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Economic Growth: The Ultimate Bird's-Eye View

J. Bradford DeLong
U.C. Berkeley

The Biggest Picture

Neoclassical economists like to make the heroic and not very well-justified assumption that at the broadest level the income paid to a factor of production--labor, capital, or natural resources--roughly corresponds to that factor's marginal contribution to increasing output. This means that we can write a simple equation for the growth rate of total production y in terms of the growth rates of labor n, of capital k, of natural resources r; of the income shares s_l, s_k, and s_r of labor, capital, and resources, and a residual factor α that captures technological inventions and innovations, improvements in business, sociological, and political organization, and other improvements in efficiency:

(1) $y = (s_l)n + (s_k)k + (s_r)r + \pi$

Over any long enough period the growth rates of output and capital will be very close. And for the world as a whole the growth of resource stocks r is zero--they contribute only as better technology enables better access to them. So we can transform our equation and solve it for π--what we economists call the rate of total factor productivity growth:

(2) $\pi = (1-(s_k))y - (s_l)n$

Table 1: Longest-Run Economic Growth

Year	Population	Income
-8000	5	$500
0	170	$500
1500	500	$500
1800	750	$600
1900	1500	$1200
2007	6300	$7000

Period	Real GDP Growth	TFP Growth (1)
-8000-0	0.04%	0.01%
0-1500	0.07%	0.02%
1500-1800	0.2%	0.09%
1800-1900	1.38%	0.89%
1900-2007	3.38%	2%

Malthusian stagnation

Toward a human world?

Where is the innovation?

Then if we are willing to make heroic and unjustified assumptions about the level of worldwide economic activity we can arrive at the accompanying made-up table for an assumed capital share s_k=0.3 and an assumed resources share s_r=0.2.

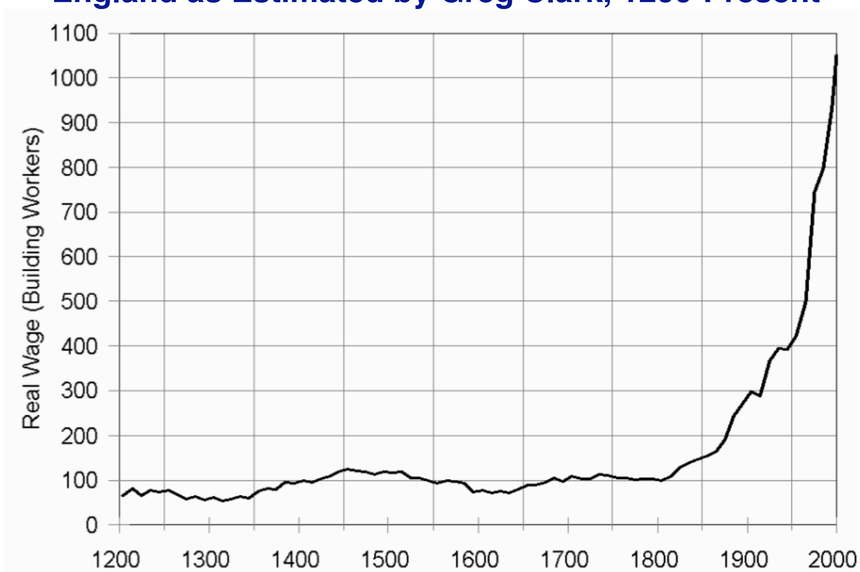
These are made-up numbers—I would not care to defend a single one of them. And this table is at an inappropriate level of aggregation, for such a global bird's-eye view misses some crucial and vitally important distinctions. Much of the increase in π across 1650 is the result of processes confined to the North Atlantic: the commercial revolution. All of the acceleration in π across 1800 is due to a fundamental sea-change in that quarter of the world economy that was the North Atlantic—the industrial revolution--and only in the North Atlantic. It is the twentieth century, not the nineteenth, was to see the spread of industrialization away from its origin points and across the globe.

Pre-Industrial Poverty

Nevertheless, this table teaches a very important lesson: economies in the long ago were very different from our economy of today. For one thing, for 95% of the time since the invention of agriculture economies were *Malthusian:* improvements in productivity and technology showed up not as increases in average standards of living but as increases in population levels at a roughly constant standard of living. For a second, in the long-long ago the pace of invention and innovation can most optimistically be described as *glacial*—two hundred years to achieve the pace of relative change that we see in twelve months. For a third, arithmetic tells you that in the long-long ago the overwhelming majority of those who are or become well-off have either held on to what their parents bequeathed them or proven successful in zero- or negative-sum redistributional struggles—rather than having found or placed themselves at a key chokepoint of positive-sum productive processes.

Why do we think that the numbers in this table are roughly the right ones?

Figure 1: Real Wages of Construction Workers in England as Estimated by Greg Clark, 1200-Present



Source: Gregory Clark (2007), *A Farewell to Alms; A Brief Economic History of the World* (Princeton: 0691121354).

For years since 1800 we actually have official and semi-official economic statistics—although with all the problems of interpreting them and correcting them for biases outlined by William Nordhaus.[1] Before 1800 our information is much more scattered

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[1] William Nordhaus (1997), "Do Real-Output and Real-Wage Measures Capture Reality? The History of Lighting Suggests Not," in Timothy Bresnahan and

but is equally compelling. We have the long-run demography fairly well nailed down—or at least guesstimated down.[2] Global human populations grew from perhaps 5 million at the time of the Neolithic Revolution—the discovery of settled agriculture and animal husbandry—some ten thousand years ago to perhaps 170 million in year zero (an annual rate of population growth of 0.04% per year); roughly tripled between the year zero and 1500 (an annual rate of population growth of 0.07% per year); grew by perhaps fifty percent in the next three centuries to 1800 (an annual rate of population growth of 0.14% per year); doubled in the century to 1900 (a rate of growth of 0.69% per year); and since the start of the twentieth century has more than quadrupled, growing to our current population of roughly 6.3 billion (an annual rate of population growth of 1.3% per year).[3]

Income levels are somewhat harder. We do, however, have calculations of real wage levels across long eras of time like that compiled by Greg Clark[4] in the figure above. Whenever we look across substantial eras of time before the industrial revolution, we find prolonged periods of near-stasis in real wages around a level

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Robert Gordon, eds. (1997), *The Economics of New Goods* (Chicago for NBER: 0226074153), pp. 29-66.

[2] See Massimo Livi-Bacci and Carl Ipsen (1997), *A Concise History of World Population* (Blackwell: 0631204555); Joel E. Cohen (1995), *How Many People Can the Earth Support?* (Norton: 0393314952).

[3] It looks as though this recent population explosion is almost over. Current projections of the demographic transition in progress have the globe's population peaking between 9 and 10 billion around 2050. See Wolfgang Lutz, Warren Sanderson and Sergei Scherbov (2001), "The End of World Population Growth," *Nature* 412 (August 2), pp. 543-5 <http://tinyurl.com/dl20090120>.

[4] Gregory Clark (2007), *A Farewell to Alms; A Brief Economic History of the World* (Princeton: 0691121354).

that we would characterize as very, very low. Second, we have the long-run biomedical studies of Rick Steckel and many others.[5] We can use Steckel's estimates of the relationship between height and income found in a cross-section of people alive today and evidence from past burials to infer what real incomes were in the past.

Table 2: Rick Steckel's Estimates of the Relationship Between Height and Income

Per Capita Income (1985 U.S. $)[a]	Boys Aged 12	Girls Aged 12	Adult Men	Adult Women
1000	137.4	138.3	163.2	151.8
2000	140.8	141.7	166.0	154.6
3000	142.8	143.7	167.6	156.2
4000	144.2	145.1	168.7	157.3
5000	145.3	146.2	169.6	158.2
6000	146.2	147.1	170.4	158.9
8000	147.6	148.5	171.5	160.1
10000	148.7	149.6	172.4	161.0
12000	149.6	150.5	173.1	161.7

Source: Richard Steckel (1995), "Stature and the Standard of Living," *Journal of Economic Literature* 33:4 (December), pp. 1903-40.

The conclusion is inescapable: people in the preindustrial past were short—very short—with adult males averaging some 63 inches compared to 69 inches either in the pre-agricultural Mesolithic or today. Therefore people in the pre-industrial past were poor—very poor. If they weren't very poor, they would have fed their children more and better and their children would have grown taller. And they were malnourished compared to us or to

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[5] Richard Steckel (2008), "Heights and Human Welfare: Recent Developments and New Directions" (NBER Working Paper 14536) <http://www.nber.org/papers/w14536.pdf>; Richard Steckel (2003), "What Can Be Learned From Skeletons That Might Interest Economists, Historians, And Other Social Scientists?" *American Economic Review* 93:2 (May), pp. 213-220.

their pre-agricultural predecessors: defects in their teeth enamel, iron-deficient, skeletal markers of severe cases of infectious disease, and crippled backs.[6]

Pre-industrial poverty lasted late. Even as of 1750 people in Britain, Sweden, and Norway were four full inches shorter than people are today—consistent with an average caloric intake of only some 2000 calories per person per day, many of whom were or were attempting to be engaged in heavy physical labor.[7] And societies in the preindustrial past were stunningly unequal: the upper classes were high and mighty indeed, upper class children growing between four and six inches taller than their working-class peers.[8] Moreover, there are no consistent trends in heights between the invention of agriculture and the coming of the industrial age. Up until the eve of the industrial revolution itself, the dominant human experience since the invention of agriculture had been one of poverty so severe as to produce substantial malnutrition and stunted growth.

It is this experience that makes Jared Diamond conclude that the invention of agriculture was the worst mistake ever made by the human race.

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[6] Jared Diamond (1987), "The Worst Mistake in the History of the Human Race" *Discover* (May), pp. 64-6 <http://tinyurl.com/dl20090112d>.

[7] Robert Fogel (1994), "Economic Growth, Population Theory, and Physiology: The Bearing of Long-Term Processes on the Making of Economic Policy," *American Economic Review* 84:3 (June), pp. 369-95.

[8] Roderick Floud, Kenneth Wachter, and Annabel Gregory (1990), *Height, Health, and History: Nutritional Status in the United Kingdom, 1750-1980* (Cambridge University: 0521303141).

Note that things were different for the elite. The lifestyles of the rich and famous in the late eighteenth century on the eve of the industrial revolution were almost certainly much better than the lifestyles of their counterparts back in the really old days, between the discovery of agriculture and the invention of printing. Suppose you live in the second millennium BC and are godlike Agamemnon, king of men, glorious son of Atreus, high king of Mycenae and lord of the Akhaians by land and sea. You are well-fed, well-housed for your time (although cold in the winter) and well-clothed (although flea-bitten and lousy). But what can you do? You can feast. You can drink. You can admire some of the (few) pretty objects that you have taken or that have been given to you. You can hunt. You can fight. You can gossip. You can sit by the fire at night and listen to one of the few songs that have been composed and remembered. You can wonder what if any songs will be composed and remembered about you. You can rape young captured Trojan women, thus angering Apollo of Delos, lord of the bow, who then strides down furious from Olympus, and with a face as dark as night smites your warriors with his plague-arrows…[9] But that is pretty much it. Starting from this 1300 BC benchmark the efflorescence of the technologies of culture and comfort up to 1800 is remarkable. But the evidence from heights tells us that this efflorescence had relatively little impact on your material standard of living if you were not part of the elite and thus had to worry about getting enough food to not be hungry, enough clothing to not be cold, and enough shelter to not be wet.

Why didn't working-class standards of living improve in the long years up until the very eve of the industrial revolution?

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[9] Homeros (n.d.), *Iliad* (Khios) <http://tinyurl.com/dl20090120a>.

Modeling Economic Growth

Let us back up and spend a little time modeling economic growth.

Economists begin to analyze long-run growth by building a simple, standard model of economic growth—a *growth model*. This standard model is also called the Solow model, after Nobel Prize-winning M.I.T. economist Robert Solow.[10] The second thing economists do is to use the model to look for an *equilibrium*--a point of balance, a condition of rest, a state of the system toward which the model will converge over time. Once you have found the equilibrium position toward which the economy tends to move, you can use it to understand how the model will behave. If you have built the right model, this will tell you in broad strokes how the economy will behave.

In economic growth economists look for the *steady-state balanced-growth equilibrium*. In a steady-state balanced-growth equilibrium the capital intensity of the economy is stable. The economy's capital stock and its level of real GDP are growing at the same proportional rate. And the capital-output ratio--the ratio of the economy's capital stock to annual real GDP--is constant.

The first component of the model is a *behavioral relationship* called the *production function*. This behavioral relationship tells us how the productive resources of the economy—the labor force, the capital stock, and the level of technology that determines the efficiency of labor—can be used to produce and determine the level of output in the economy. The total volume of production of the goods and services that consumers, investing businesses, and the government wish for is limited by the available resources. Tell the production function what resources the economy has available,

[10] Robert Solow (1956), "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics* 70:1 (February), pp. 65-94 <http://tinyurl.com/dl20090120b>.

and it will tell you how much the economy can produce. We are going to use a simple and analytically convenient production function:

(3) $$Y = A\left(K^{s_k}\right)\left(R^{s_r}\right)\left(L^{s_l}\right)$$

The economy's total level of production is equal to a total factor productivity term A times the product of:

- the economy's capital stock K raised to the power s_k equal to capital's share of income.
- the economy's resource stock R raised to the power s_r equal to the resource owners' share of income.
- the economy's labor force L raised to the power s_l equal to labor's share of income.

And the three income shares add up to one: together, capital-owners, resource-owners, and workers receive all the income produced in the economy.

(4) $$s_k + s_r + s_l = 1$$

If we take the natural log of the production function we get:

(5) $$\ln(Y) = \ln(A) + s_k \ln(K) + s_r \ln(R) + s_l \ln(L)$$

And then writing y for the proportional growth rate—the log change—of Y, k for the proportional growth rate of K, r for the proportional growth rate of R, n for the proportional growth rate of L, and π for the proportional growth rate of A gets us the equation (1) with which we started.

No economist believes that there is, buried somewhere in the earth, a big machine that forces the level of output per worker to behave exactly as calculated by the algebraic production function above.

Instead, economists think that this Cobb-Douglas production function above is a simple and useful approximation. The true process that does determine the level of output per worker is an immensely complicated one: everyone in the economy is part of it. And it is too complicated to work with. Writing down the Cobb-Douglas production function is a breathtakingly large leap of abstraction. Yet it is a useful leap, for this approximation is good enough that using it to analyze the economy will get us to approximately correct conclusions.

Assume that the growth rates of technology and resources are constant at π and 0, respectively—resources are just there. They do not grow. Further assume that a constant proportional share S for savings of output Y is saved each year and invested to add to the capital stock. Further assume that a fraction δ of the capital stock wears out or is scrapped each period. Thus the proportional rate of growth of the capital stock k is:

(6) $$k = \frac{1}{K}\frac{dK}{dt} = S\left(\frac{Y}{K}\right) - \delta$$

Consequences of Malthusian Population

Let us also assume that the richer is the economy, the faster is population and labor-force growth:

(7) $$n = \gamma(1-S)\left[\left(\frac{Y}{L}\right) - C^{sub}\right]$$

with there being some level C^{sub} of consumption per worker—what is left after resources for invesment have been deducted—that counts as “subsistence”: at which population growth is zero. And let’s look for an equilibrium in this model—an equilibrium which the capital-output ratio is constant and output per worker is constant, Thus output, the capital stock, and the labor force will all be growing at the same constant rate n—and resources will be growing at the constant growth rate of 0.

Substituting into equation (1):

(8) $n = (s_l)n + (s_k)n + \pi$

(9) $(1 - s_l - s_k)n = \pi$

(10) $n = \dfrac{\pi}{1 - s_l - s_k}$

This is the equation from which the productivity growth rates in Table 1 were calculated.

We can then substitute:

(11) $\dfrac{\pi}{1 - s_l - s_k} = \gamma(1 - S)\left[\left(\dfrac{Y}{L}\right) - C^{sub}\right]$

and derive:

(12) $\left(\dfrac{Y}{L}\right)^* = \dfrac{1}{1 - S}\left[Y^{sub} + \dfrac{\pi}{\gamma(1 - s_l - s_k)}\right]$

where the * denotes that the left-hand side is a balanced-growth equilibrium value. This is not only an equilibrium of the model, it is the only equilibrium of the model for the production function (5), the capital accumulation equation (6), constant resources, and population growth function (7), and a constant rate of total factor productivity growth π. If Y/L is above the level given in (12), then population growth is faster than $\pi/(1-s_l-s_k)$ and output per worker falls. If Y/L is below the level given in (12), then population growth is faster than $\pi/(1-s_l-s_k)$ and output per worker rises. In either case, output per worker converges back toward its value given by (12).

We now have some insight into the long period of relative stagnation in living standards, for it is clear how the model works. Change the propensity to save S and in the long run you don't change the level of consumption per capita. Change the rate of technological progress π and you change the level of consumption per capita—but only by a little bit. Find more natural resources in a sudden discrete leap—and in the long run your level of consumption per capita goes back to what it was before. All of these changes have large effects on the size of the human population. But they don't have large effects on standards of living. To affect standards of living in this model you have to affect the "subsistence" consumption level C^{sub}.

How did we escape from this Malthusian trap—in which most of the human race was held between 6000 BC and 1650 or so? That's a complex and still poorly understood story…

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