Finger Exercise: Secret History of the Industrial Revolution

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September 7, 2009

As Gregory Clark wrote in his "The Secret History of the Industrial Revolution":

> [T]he effects of individual technical advances on aggregate productivity depend crucially on such accidental factors as the size of the sector affected and the price elasticity of demand…. [S]ome new idea leads to a long period of productivity advance in an industry as the consequences of the new technique are played out. If demand is price inelastic then reductions in prices created by the early phase of a technological advance will limit or even reduce the share of expenditure on the good, so reducing the general productivity gains from further advances. Advances in cotton textiles in the Industrial Revolution had big impacts because textiles were a substantial share of expenditure by the 1760s and demand was price elastic....
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> Suppose that prior to the Industrial Revolution innovations were occurring randomly across various sectors of the economy—innovations such as guns, spectacles, books, clocks, painting, new building techniques, improvements in shipping and navigation—but that just by chance all these innovations occurred in areas of small expenditure and/or low price elasticities of demand. Then the technological dynamism of the economy would not show up in terms of output per capita or in measured productivity.
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> Thus... consider the introduction of the printed book by Gutenberg in 1445… in the period where we can find no evidence of aggregate

> productivity growth, at least in England.... Output per worker increased by roughly 30 fold from manuscript production in the fourteenth century till the early nineteenth century... greater than the productivity advances achieved in the cotton textile industry over the Industrial Revolution period.... But the impact of these productivity gains in printing on the economy as a whole was unmeasurably small because the share of the economy devoted to printing always remained small... in 1851 only 0.8% of the population was employed in the paper making and printing businesses...

Let us do some finger exercises on this theme. Let's consider a series of technological innovations that advance productivity by a factor g in a sector that initially starts with an expenditure share k and that has a price-elasticity of demand b.

The expenditure share for this sector S will then be given by:[1]

$$S = kP^{1-b}$$

We know that the change in economic wealth W depends on the change in the price P of goods produced by the technologically-progressive sector according to:

$$\frac{dW}{W} = S\left(-\frac{dP}{P}\right)$$

We can then do a little math:

$$\frac{dW}{W} = kP^{1-b}\left(-\frac{dP}{P}\right)$$

$$\frac{dW}{W} = -kP^{-b}dP$$

$$d\ln(W) = -kP^{-b}dP$$

After which we will seek to turn our indefinite integral into a definite one:

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[1] Note that we are thus restricting $b \leq 1 - \ln(k)/\ln(g)$ in order to keep expenditure shares below 1. This is a small sacrifice to pay for simple closed-form solutions.

$$\ln(W) - \ln(W_0) = \int_{1/g}^{1} -kP^{-b}\,dP$$

$$\ln(W) - \ln(W_0) = \frac{k}{1-b}\left[1-(1/g)^{1-b}\right]$$

$$\frac{W}{W_0} = \exp\left(\frac{k}{1-b}\left[1-(1/g)^{1-b}\right]\right)$$

Table 1: Relative Changes in Economic Wealth dW as a Function of Initial Expenditure Share k, Demand Elasticity b, and Productivity Multiplication Factor g

k	b	g	dln(W)	dW
0.1	0.25	2	0.05	0.06
0.1	0.25	4	0.09	0.09
0.1	0.25	16	0.12	0.12
0.1	0.25	128	0.13	0.14
0.1	0.25	1024	0.13	0.14
0.1	0.50	2	0.06	0.06
0.1	0.50	4	0.10	0.11
0.1	0.50	16	0.15	0.16
0.1	0.50	128	0.18	0.20
0.1	0.50	1024	0.19	0.21
0.1	1.00	2	0.07	0.07
0.1	1.00	4	0.14	0.15
0.1	1.00	16	0.28	0.32
0.1	1.00	128	0.49	0.62
0.1	1.00	1024	0.69	1.00
0.1	1.33	2	0.08	0.08
0.1	1.33	4	0.18	0.19
0.1	1.33	16	0.46	0.58
0.1	1.33	128	1.21	2.36
0.1	1.33	1024	2.72	14.17

Table 1 shows the effect of sample parameter values. When demand is inelastic—a value of b of 0.25—even though the initial expenditure share k is a fairly large 10%, nearly half of the increase in wealth from a near-infinite multiplication in sector productivity comes from the first doubling of productivity. By the time productivity in the sector has multiplied tenfold the expenditure share has fallen by a factor of six. At a price elasticity of one—so that the expenditure share is constant at 10%—each doubling in productivity adds 7%—the expenditure share times the natural log increase in productivity—to wealth. And when we push the price elasticity of demand above one, then each doubling of productivity has a larger effect on wealth until the expenditure share hits one and the model fails to apply.

This is Clark's point: the same raw technological impetus applied to sectors with very different elasticities of demand and initial expenditure shares will have very different aggregate economic impacts.

But Clark's point is incomplete. As long as expenditure shares are limited, even extraordinary rates of invention and innovation *in any one sector* have a limited impact on overall economic growth. Consider lines 15-20 in Table 1 above: a sector with a healthy 10% expenditure share, not subject to satiation, and yet at thousand-fold multiplication of productivity in the sector leads to a mere doubling of aggregate wealth.

To truly get modern economic growth—not just one doubling of wealth but repeated, ongoing doublings—no narrowly-focused set of inventions and innovations will suffice. You need broad-based economic growth produced by constantly revolutionizing technologies in most if not all of the economy.

777 words: September 7, 2009