Econ 115: Final Problem Set
Due Monday, December 14, 2009

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Fall 2009

The proportional rate of growth (in percent per year) of the capital stock in a modern economy depends on two factors: the depreciation rate δ with which old capital wears out and rusts away, and the ratio of investment I to the level of the capital stock K. Let's use lower-case italic k for this proportional rate of growth. And let's set investment I equal to the share of production s that is saved and invested times the level of production Y.

1) Write down an equation for k. Justify the equation you have written down.

Lower-case italic y, the proportional rate of growth of production in a modern economy, is equal to the share of income produced by capital α times k, the proportional rate of growth of the capital stock, plus $(1-\alpha)$ times the sum of l, the proportional rate of growth of the labor force, and e, the rate of improvement in the efficiency of labor due to inventing or copying better forms of organization and modes of technology.

2) Write down an equation for y. Justify the equation you have written down.

The proportional change in a modern economy's output-capital ratio Y/K is the difference between y, the proportional growth rate of output, and k, the proportional growth rate of the capital stock.

3) Write down an equation for the proportional change in a modern economy's output-capital ratio Y/K. Simplify it. Justify your equation.

4) At what level of the output-to-capital ratio will it be stable—will it be neither rising nor falling? Justify your answer.

Hereafter, assume that the output-to-capital ratio is at that level such that it tends to neither rise nor fall.

5) What is the proportional rate of growth of output per capita in this economy?

6) What happens to the steady-state output-to-capital ratio in a modern economy if the population growth rate *l* falls due to a successful demographic transition?

7) If you have successfully worked your way through (1) through (6), you will have arrived at the conclusions that:

- An economy with a lower population growth rate will tend to be more capital intensive—have a lower ratio of output-to-capital—and so tend to be richer, because each worker will have more capital equipment at his or her disposal to amplify productivity.
- An economy's growth rate of output per capita will in the long run depend overwhelmingly on its *e*—on the speed with which it improves its efficiency of labor as it invents or borrows better forms of organization and of technology

Write a paragraph about how, in your opinion, these two conclusions do or do not illuminate the convergence or failure thereof of the globe's economies in the relatively peaceful years since World War II.