IAS 107: Spring 2011: Problem Set 2

Due at the start of lecture on Th Feb 3

1. Jean Baptiste Say in 1803 claimed that because nobody makes anything without intending to use it or sell it, and nobody sells anything without intending to buy something else, that there could be no general shortage of demand in an economy-- that there could be a planned excess of supply of some commodities, but it would be balanced by a planned excess of demand of some other commodities. Was he wrong? Why was he wrong?

   Jean-Baptiste Say did not think of the possibility that people in aggregate might want to buy (or sell) financial assets. You can have a shortage of demand for currently-produced goods and services if there is an economy-wide excess demand for financial assets, like cash to hold in your pocket, savings vehicles like bonds to transfer your purchasing power from the present into the future, or safe assets like the bonds of solvent governments so that you can sleep more easily.

2. About how many people were unemployed in the United States back in 2007? How does this compare to the number unemployed today?

   About 7 million people were unemployed in the United States back in 2007. About 15 million people are unemployed today.

3. Roughly, what is the gap between real per capita GDP in Canada today, real per capita in Brazil, and real GDP per capita in Zimbabwe?

   According to gapminder.org, real GDP in Zimbabwe in 2005 was $538. Real GDP in Brazil in 2008 was $9,682, and real GDP in Canada in 2007 was $36,225.

4. Consider an economy with the production function: $Y=K^{\alpha}(EL)^{(1-\alpha)}$ on its steady-state balanced-growth path:

   a. Suppose α = 1/2, E=1, L=100, and K=64; what is output per worker Y/L?
   b. Suppose α = 1/2, E=3, L=196, and K=49; what is output per worker Y/L?
   c. If both capital K and labor L triple, what happens to total output Y?
   d. Holding E=1, suppose that capital per worker increases from 4 to 8 and then from 8 to 12. What happens to output per worker?

a. Plugging in the values to the formula, and remembering that raising a number to the ½ power is the same as taking its square root: Y = 8 x 10 x 1 = 80; L = 100; Y/L = 0.8
b. Plugging in the values to the formula, and remembering that raising a number to the ½ power is the same as taking its square root: Y = 7 x 14 x sqrt (3) = 169.74; L = 196; Y/L = 0.86
c. Output triples—the production function was designed that way.
d. As capital per worker doubles output per worker grows by the square root as much--by sqrt(2) - 1 or 41.4%, from 2 to 2.83. As capital per worker grows by 50%, output per worker grows by sqrt(1.5) -1 or 22.5%, from 2.83 to 3.46.

5. Consider an economy with the production function: $Y=K^{\alpha}(EL)^{(1-\alpha)}$ on its steady-state balanced-growth path:

   a. Suppose α = 1/3, E=1, L=100, and K=64; what is output per worker Y/L?
   b. Suppose α = 1/3, E=3, L=196, and K=49; what is output per worker Y/L?
   c. If both capital K and labor L double, what happens to total output Y?
   d. Holding E=1, suppose that capital per worker increases from 1 to 4 and then from 4 to 9. What happens to output per worker?

   a. Plugging in the values to the formula, and remembering that raising a number to the ⅓ power is the same as taking its cube root: Y = 64^(⅓) x 100^(⅔) = 86.2; L = 100; Y/L = 0.862
   b. Plugging in the values to the formula, and remembering that raising a number to the ⅓ power is the same as taking its cube root: Y = 49^(⅓) x 196^(⅔) X 3^(⅔) = 256.83; L = 196; Y/L = 1.31
   c. Output doubles—the production function was designed that way
   d. As capital per worker doubles, output per worker grows by the cube root as much--by cube(4) - 1 or 58%, from 1 to 1.58. As capital per worker grows by 50%, output per worker grows by cube(9/4) -1 or 31% from 1.58 to 2.08

6. Suppose that an economy's production function is $Y=K^{\alpha}(EL)^{(1-\alpha)}$ with α=0.5; suppose further that the savings rate s is 40% of GDP, that the depreciation rate δ is 4% per year, the population growth rate n is 0% per year, and the rate of growth g of the efficiency of the labor force is 2% per year.

   a. What is the steady-state balanced-growth capital-output ratio?
   b. How fast does output per worker grow along the steady-state balanced-growth path?
   c. How fast does total output grow along the steady-state balanced-growth path?

   a. First, calculate the capital intensity in steady-state balanced growth using the formula: s/(n+g+d) = 40/(4+0+2) = 6.7. The capital-output ratio is 6.7.
   b. Recall that in steady-state balanced-growth output per worker grows at the same speed as the efficiency of labor g. The efficiency of labor g grows at 2%

per year along the balanced-growth steady-state path: output per worker, the capital stock per worker, and the efficiency of labor are then all growing at the same rate--balanced growth. Why? Well, if things worked out differently we wouldn't be calling the right thing "efficiency of labor growth," would we?
c. Total output grows at the sum of the growth rates of output per worker and of the labor force. Since the labor force is not growing, total output is growing at the 2% per year of output per worker as well as well.

7. Suppose that all variables are the same as in problem 6 save the production function, which instead has α=0.8; how would your answers to 6a, b, and c be different? Why would your answers are different?

a. s/(n+g+d) = 40/(4+0+2) = 6.7. The capital-output ratio is 6.7.
b. Output per worker grows at the same speed as the efficiency of labor g. The efficiency of labor g grows at 2% per year along the balanced-growth steady-state path: output per worker, the capital stock per worker, and the efficiency of labor are then all growing at the same rate--balanced growth. Why? Well, if things worked out differently we wouldn't be calling the right thing "efficiency of labor growth," would we?
c. Total output grows at the sum of the growth rates of output per worker and of the labor force. Since the labor force is not growing, total output is growing at 2% per year as well.
d. You see that the answers don't change. Why don't they change? Why should they? The diminishing-returns parameter α has nothing to do with the capital-output ratio--that is s/(n+g+d). And the diminishing returns parameter has nothing to do with the growth rate--that is g. What the diminishing returns parameter does determine is the level of output per worker Y/L at any point in time.

8. Botswana: In the 1950s Botswana's savings rate averaged 6% of GDP. In 1960 Botswana's level of GDP per capita was $900 of today's dollars per year. Since 1960 Botswana's savings rate has averaged 30% of GDP. Today Botswana's level of GDP per capita is $15000 per year. Assume that the diminishing-returns parameter α in our production function for Botswana is 0.5, that Botswana's population growth rate n has been constant at 2% per year, and that its depreciation rate δ has been constant at 4% per year. Assume that Botswana was back in 1960 on its old steady-state balanced-growth path (for an s=0.06) and is now on its new steady-state balanced-growth path (for an s=0.30)

a. Suppose there had been no growth in the efficiency of labor in Botswana between 1960 and 2011, what do you predict that the level of GDP per capita would be in Botswana today?
b. How fast has the efficiency of labor grown in Botswana over the past 50 years?
c. What was the value of the efficiency of labor in Botswana in 1960?
d. What is the value of the efficiency of labor in Botswana today?

a. Quintupling the savings rate means that the capital-output ratio today is 5 times its value as of 1960. With the diminishing-returns parameter α of ½, each increase in the capital-output ratio produces a proportionate increase in output per worker. Thus if there had been no efficiency of labor growth GDP per worker in Botswana today would be 5 times its 1960 level.
b. Instead of being 5 times its 1960 level, GDP per capita today is 16 ⅔ times its 1960 level--that is an additional growth factor of 3 ⅓. For a quantity to grow by a factor of 3 ⅓ in 51 years requires a growth rate of 2.39% per year. That has been the growth rate of the efficiency of labor in Botswana.
c. In 1960 Botswana had a capital-output ratio of s/(n+g+d) = .06/(.02+.0238+4) = 0.716. Since $900 = 0.716 x E, E was then $1,257/year.
d. Today Botswana has a capital-output ratio of s/(n+g+d) = .3/(.02+.0238+4) = 3.58. Since $15000 = 3.58 x E, E is now $4,200/year.

9. Zambia: In the 1950s Zambia's savings rate averaged 24.5% of GDP. In 1960 Zambia's level of GDP per capita was $1800 of today's dollars per year. Since 1980 Zambia's savings rate has averaged 24.5% of GDP. Today Zambia's level of GDP per capita is $1300 per year. Assume that the diminishing-returns parameter α in our production function for Zambia is 0.5, that Zambia's population growth rate n has been constant at 3% per year, and that its depreciation rate δ has been constant at 4.64% per year.

a. Suppose there had been no growth in the efficiency of labor in Zambia between 1960 and 2011, what do you predict that the level of GDP per capita would be in Zambia today?
b. How fast has the efficiency of labor grown in Zambia over the past 50 years?
c. What was the value of the efficiency of labor in Zambia in 1960?
d. What is the value of the efficiency of labor in Zambia today?

a. With no change in the capital-output ratio and no growth in the efficiency of labor, Zambian GDP per capita today should be the same as it was back in 1960: $1800/year.
b. Since GDP per capita is now $1300/year, the efficiency of labor in Zambia must have shrunk at a rate of (1300/1800)^(1/51)-1 or -0.64%./year
c. With n+g+d of 7% and a savings-investment share s of 24.5% Zambia's capital-output ratio has been constant at 3.5. That means that the efficiency of labor in Zambia in 1960 was $514/year
d. The efficiency of labor in Zambia today is $371/year

10. France: Since 1946 French population growth (including illegal immigration) has been constant at about 1% per year and France has had a savings share of 25% of GDP. Today France has a GDP per capita level of about $35,000 per year. The rate of growth of the efficiency of labor in France since the end of World War II in France has been constant at about 2% per year. Assume that France is today on its steady-state balanced-growth path.

a. If France remains on its current steady-state balanced-growth path, what will GDP per capita be in France in 2050?
b. If France remains on its current steady-state balanced-growth path, what will GDP per capita be in France in 2100?
c. What would France's level of GDP per capita have been back in 1946 if it had then been on today's steady-state balanced-growth path?
d. In fact, France's level of GDP per capita back in 1946 was about $3,000 per year even though its efficiency of labor has grown at 2% per year since the end of World War II. Why do you think its level back then was so low?

a. French GDP per capita is growing at 2% per year. From today's level of $35000 per year, growth at 2% per year for 39 more years will give it a level of 35000 x (1.02)^39 = $75,766/year
b. French GDP per capita is growing at 2% per year. From today's level of $35000 per year, growth at 2% per year for 89 more years will give it a level of 35000 x (1.02)^39 = $203,931/year
c. If French GDP per capita had been growing at 2% per year since 1945, GDP per capita at the end of World War II would have been 35000 x (1.02)^(-65) = $9855/year.
d. France's capital-output ratio must have been very low at the end of World War II—because of low investment during the Great Depression, the Nazis, our bombs, and then the Liberation campaign.

11. Japan has had a very high savings rate and a high growth rate of output per worker over the past half century, starting from an initial post-WWII very low level of capital per worker. What does the analysis of chapter 4 suggest about Japan's ability to sustain a higher growth rate than other industrial countries?

Some of the boost to Japanese prosperity comes from rebuilding the capital stock after wartime destruction--if France is a good analogy, that accounts for a quadrupling of growth. Little can be accounted for by a falling rate of population growth. So the rest of Japan's economic boom must be driven by the application to production of new and better technologies that raised the efficiency of labor.

12. Suppose that environmental regulations lead to a diversion of investment spending from investments that boost the capital stock to investments that replace polluting with less-polluting capital. In our standard growth model, what would be the consequences of such a diversion for the economy's capital-output ratio and for its balanced-growth path? Would it make sense to say that these environmental regulations diminished the economy's wealth?

Such a diversion shows up as a reduction in the steady-state balanced-growth capital-output ratio, and as a level of GDP per capita less than it would otherwise have been. But it makes no sense to say that this diminishes the

economy's wealth broadly construed--unless, of course, the pollution cleanup money is wasted.