IAS 107: Spring 2011: Problem Set 3

Due at the start of lecture on Th Feb 10

1. In the 1950s, South Korea had a savings-investment share of GDP of 10%. In 1960, South Korea had a GDP per worker level of $2000 (at 2010 prices, in international dollars). Since 1960 South Korea's savings-investment share of GDP has averaged 27.5%. Today South Korea has a GDP per worker level of $40,000. In the 1950s, South Korea's population growth rate averaged 3% per year. Since 1960 South Korea's population growth rate has averaged 1% per year. Assume that the depreciation rate on the capital stock has been constant at 5%. Assume that the diminishing-returns parameter in the production function $\alpha=0.5$. Assume that the growth rate of South Korea's efficiency of labor was 0 in the 1950s, and has been constant at some positive value g since. Assume that South Korea in 1960 was in its steady-state balanced-growth path, and is today on its steady-state balanced growth path.

   a. What was South Korea's efficiency of labor E in 1960?
   b. Suppose the rate of growth of the efficiency of labor in South Korea since 1960 has averaged 6% per year. What would be the efficiency of labor in South Korea today?
   c. Suppose the rate of growth of the efficiency of labor in South Korea since 1960 has averaged 5% per year. What would be the capital-output ratio in South Korea today?
   d. Suppose the rate of growth of the efficiency of labor in South Korea since 1960 has averaged 5% per year. What would be the level of output per worker in South Korea today?
   e. Do you think the average growth rate of the efficiency of labor in South Korea since 1960 has been faster or slower than 5%. Why?

a. With a savings share of 10% in the 1950s and with investment requirements of 8% in the 1950s, in 1960 South Korea had a capital-output ratio of 1.25. With an $\alpha = ½$ and a GDP per worker in 1960 of $2000/year, that means an efficiency of labor of $1600/year.

b. Using the compound growth equation we get $1600/year x $(1.06)^{50}$ = $29,500/year.

c. With a savings share of 27.5% and investment requirements of 11% since 1960, South Korea's capital-output ratio today would be 2.5.

d. At 5% per year compound growth we get an efficiency of labor today of $18,350/year. With $\alpha = ½$ we multiply that by the capital-output ratio of 2.5 to get current output per worker—$45,900/year.

e. Since actual output per worker is some 15% less than $45,900/year, we can estimate that actual growth in the efficiency of labor has been less than 5%/year. Dividing 15% by 50 years produces an approximate difference of 0.3%/year between actual growth and 5%/year.

2. Suppose that in question (3) were the same, but with α=2/3 rather than α=0.5. How would your answers be different?

a. With a savings share of 10% in the 1950s and with investment requirements of 8% in the 1950s, in 1960 South Korea had a capital-output ratio of 1.25. With an α =⅔ and a GDP per worker in 1960 of $2000/year, that means $Y/L = (1.25)^2 \times E$. $2000 = 1.5625 x E then gives us E = $1280/year.

b. Using the compound growth equation we get $1280/year x $(1.06)^{50}$ = $23,600/year.

c. With a savings share of 27.5% and investment requirements of 11% since 1960, South Korea's capital-output ratio today would be 2.5.

d. At 5% per year compound growth we get an efficiency of labor today of $14,700/year. With α = ⅔ we multiply that by the square of the capital-output ratio of 6.25 to get current output per worker—$91,700/year.

e. Since actual output per worker is less than half of that, we can estimate that actual growth in the efficiency of labor has surely been less than 5%/year.

3. Since 1960 South Korea's savings-investment share of GDP has averaged 27.5%. Since 1960 the United States's savings-investment share of GDP has averaged 20%. Today South Korea has a GDP per worker level of $40,000. Today the United States has a GDP per worker level of $70,000. Since 1960 South Korea's and the United States's population growth rates have both averaged 1%/year. Assume that the depreciation rate on the capital stock has been constant at 5%/year. Assume that the rate of improvement of the efficiency of labor in the United States has averaged 2% per year and the rate of growth of the efficiency of labor in South Korea has averaged 5% per year. Assume that both South Korea and the United States today are on their balanced growth paths.

a. What is the efficiency of labor in South Korea today?
b. What is the efficiency of labor in the United States today?
c. If the efficiency of labor in the United States continues to grow at its long-run trend pace of 2% per year, what is your forecast of the level of output per worker in the United States in 2100?
d. What is your forecast of output per worker in South Korea in 2100?

a. With a savings share of 27.5% and investment requirements of 11%, South Korea's current capital-output ratio is 2.5. With GDP per worker today at $40,000/year, that tells us that the efficiency of labor in South Korea today is $16,000/year.

b. With a savings share of 20% and investment requirements of 8%, the U.S. Current capital output ratio is 2.5. With GDP per worker at $70,000/year, that tells us that the efficiency of labor in the U.S. Today is some $28,000/year

c. Since output per worker grows at the same pace as the efficiency of labor for a country on its balanced growth path, we take $70,000 x $(1.02)^{90}$ = $416,000

d. As discussed in lecture, it seems likely that South Korea's efficiency of labor will stop growing at 5%/year and start growing at only 2%/year once South Korea catches up to the development level of the world economy's North Atlantic core. At that point South Korea will have a savings share of 27.5% and investment requirements of 8%, giving it a steady-state balanced-growth capital-output ratio of 3.4375. Multiplying that by the leading-edge efficiency of labor in 2100 of $166,000/year gives us an output per worker level of $572,000/year in 2100

4. Bangladesh: In 1960 annual output per worker in what was to become Bangladesh averaged $1200. Today annual output per worker in Bangladesh averages $3000. If output per worker in Bangladesh continues to grow at the average pace it has grown since 1960...

a. How long will it take Bangladesh to achieve the productivity levels that South Korea has today?
b. How long will it take Bangladesh to achieve the productivity levels that the U.S. has today?
c. If output per worker in the U.S. continues to grow at its long-run historical average rate of 2%/year, what will output per worker in the U.S. be when Bangladesh becomes as prosperous as the United States is now?

a. Bangladeshi output per worker has been growing at ln(3000/1200)/50 = .018 = 1.8% per year since 1960. If t is the number of years until it reaches where South Korea is today, the equation to solve is $40,000 = $3,000 x $(1.018)^t$, the solution to which is t = ln(40000/3000)/.018 = 144 years. So we are talking 2154.
b. With the U.S. $70,000, we have t = ln(70000/3000)/.018 = 175 years. So we are talking 2185.
c. In 2185 if U.S GDP per worker continues to grow at 2% per year we are talking 70000 x $(1.02)^{175}$ = $2,240,000/year

5. Roughly, what is the gap between real per capita GDP in Belgium today, real per capita in Indonesia, and real GDP per capita in Nigeria?

Belgium: $32,300. Indonesia: $3,800. Nigeria: $2,160