DeLong and Olney Macroeconomics 3rd Edition ch 4.4: Using the Solow Growth Model

4.4 USING THE SOLOW GROWTH MODEL

Up until now we have assumed that all the parameters of the Solow growth model are unchanging. This is false. But it allowed us to make progress.

Now let's try to make some more progress:

4.4.1 Jumps in Parameter Values

What if one or more of the parameters in the Solow growth model were to suddenly and substantially shift? What if the labor-force growth rate were to rise, or the rate of technological progress to fall?

One principal use of the Solow growth model is to analyze questions like these: how changes in the economic environment and in economic policy will affect an economy's long-run levels and growth path of output per worker Y/L.

Let's consider, as examples, several such shifts: an increase in the growth rate of the labor force n, a change in the economy's saving-investment rate s, and a change in the growth rate of labor efficiency g. All of these will have effects on the balanced- growth path level of output per worker. But only one—the change in the growth rate of labor efficiency—will permanently affect the growth rate of the economy.

We will assume that the economy starts on its balanced growth path—the old balanced growth path, the pre-shift balanced growth path. Then we will have one (or more) of the parameters—the savings-investment rate s, the labor force growth rate n, the labor efficiency growth rate g—jump discontinuously, and then remain at its new level indefinitely. The jump will shift the balanced growth path. But the level of output per worker will not immediately jump. Instead, the economy's variables will then, starting from their old balanced growth path values, begin to converge to the new balanced growth path—and converge in the standard way.

Remind yourselves of the key equations for understanding the model:

The level of output per worker is:

(4.4.1) $$\left(\frac{Y}{L}\right) = \left(\frac{K}{Y}\right)^{\left(\frac{\alpha}{1-\alpha}\right)}(E)$$

The balanced-growth path level of output per worker is:

(4.4.2) $$\left(\frac{Y}{L}\right)^* = \left(\frac{s}{n+g+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)}(E)$$

The speed of convergence of the capital-output ratio to its balanced-growth path value is:

(4.4.3) $$\frac{d(K/Y)}{dt} = -(1-\alpha)(n+g+\delta)\left(\frac{K}{Y} - \frac{s}{n+g+\delta}\right)$$

4.4.2 A Shift in the Labor-Force Growth Rate

Real-world economies exhibit profound shifts in labor-force growth. The average woman in India today has only half the number of children that the average woman in India had only half a century ago. The U.S. labor force in the early eighteenth century grew at nearly 3 percent per year, doubling every 24 years. Today the U.S. labor force grows at 1 percent per year. Changes in the level of prosperity, changes in the freedom of migration, changes in the status of women that open up new categories of jobs to them (Supreme Court Justice Sandra Day O'Connor could not get a private-sector legal job in San Francisco when she graduated from Stanford Law School even with her amazingly high class rank), changes in the average age of marriage or the availability of birth control that change fertility—all of these have powerful effects on economies' rates of labor-force growth.

What effects do such changes have on output per worker Y/L—on our mea sure of material prosperity? The faster the growth rate of the labor force n, the lower will be the economy's balanced-growth capital-output ratio s/(n + g - δ). Why? Because each new worker who joins the labor force must be equipped with enough capital to be productive and to, on average, match the productivity of his or her peers. The faster the rate of growth of the labor force, the larger the share of current investment that must go to equip new members of the labor force with the capital they need to be productive. Thus the lower will be the amount of invest ment that can be devoted to building up the average ratio of capital to output.

A sudden and permanent increase in the rate of growth of the labor force will lower the level of output per worker on the balanced-growth path. How large will the long-run change in the level of output be, relative to what would have happened had labor-force growth not increased? It is straightforward to calculate if we know the other parameter values, as is shown in Box 4.4.1.

Box 4.4.1: An Increase in the Labor Force Growth Rate: An Example:

Consider an economy in which the parameter α is 1/2, the efficiency of labor growth rate g is 1.5 percent per year, the depreciation rate δ is 3.5 percent per year, and the saving rate s is 21 percent. Suppose that the labor-force growth rate suddenly and permanently increases from 1 to 2 percent per year.

Before the increase in the labor-force growth rate, the balanced-growth equilibrium capital-output ratio was:

$$\frac{K}{Y} = \frac{s}{n+g+\delta} = \frac{0.21}{0.01+0.015+0.035} = 3.5$$

After the increase in the labor-force growth rate, the new balanced-growth equilibrium capital-output ratio will be:

$$\frac{K}{Y} = \frac{s}{n+g+\delta} = \frac{0.21}{0.02+0.015+0.035} = 3$$

Before the increase in labor-force growth, the level of output per worker along the balanced-growth path was equal to:

$$\frac{Y_t}{L_t} = \left(\frac{s}{n+g+\delta}\right)^{\alpha/(1-\alpha)} E_t = 3.5^1(E_t) = 3.5(E_t)$$

After the increase in labor-force growth, the level of output per worker along the balanced-growth path will be equal to:

$$\frac{Y_t}{L_t} = \left(\frac{s}{n+g+\delta}\right)^{\alpha/(1-\alpha)} E_t = 3^1(E_t) = 3(E_t)$$

This fall in the balanced-growth path level of output per worker means that in the long run—after the economy has converged to its new balanced-growth path—one-seventh of its *per worker* economic prosperity has been lost because of the increase in the rate of labor-force growth.

In the short run of a year or two, however, such an increase in the labor-force growth rate has little effect on output per worker. In the months and years after labor-force growth increases, the increased rate of labor-force growth has had no time to affect the economy's capital-output ratio. But over decades and generations, the capital-output ratio will fall as it converges to its new balanced-growth equilibrium level.

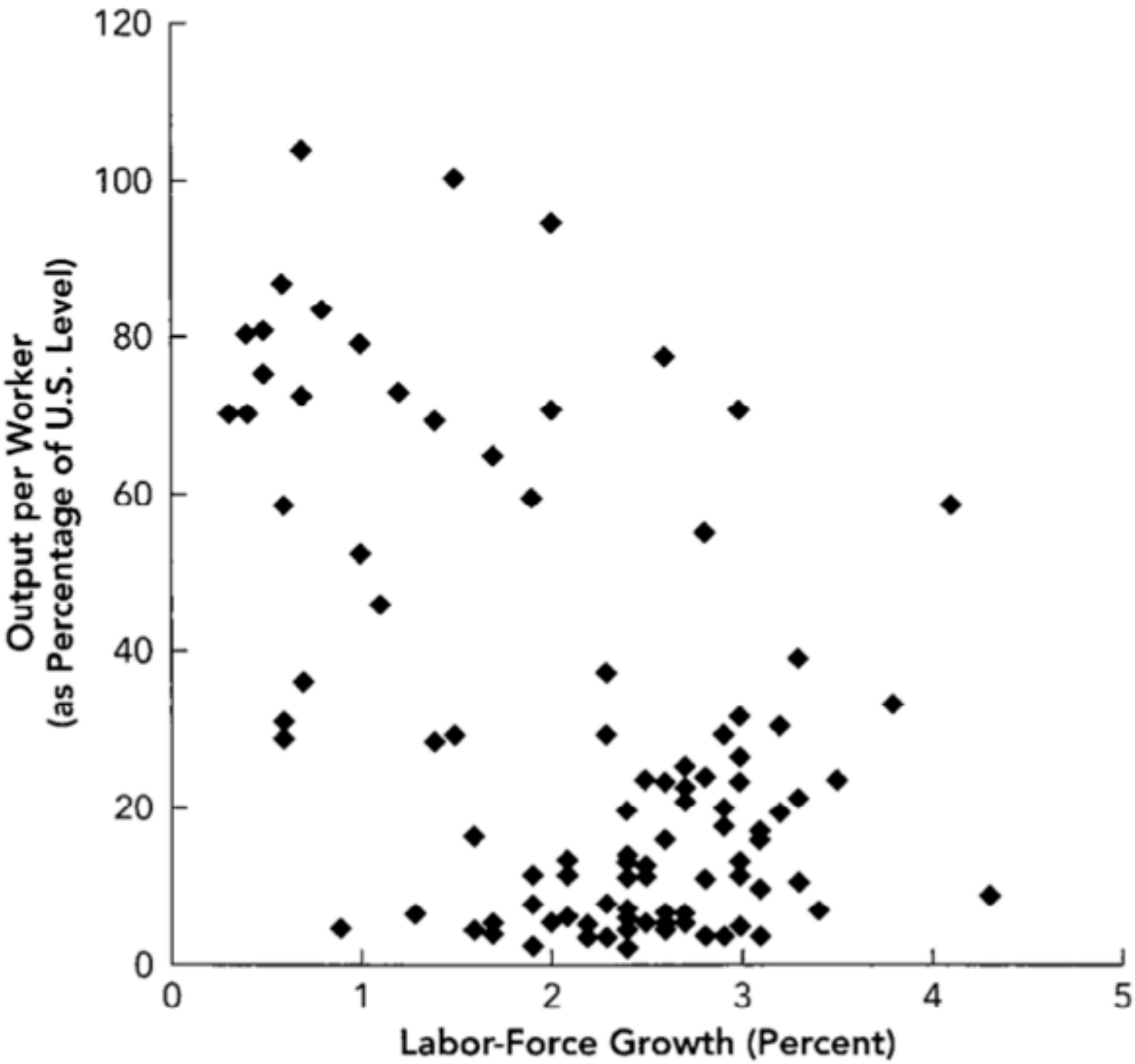
A sudden and permanent change in the rate of growth of the labor force will immediately and substantially change the level of output per worker along the economy's balanced-growth path: It will shift the balanced-growth path for output per worker up (if labor-force growth falls) or down (if labor-force growth rises). But there is no corresponding immediate jump in the actual level of output per worker in the economy. Output per worker doesn't immediately jump—it is just that the shift in the balanced-growth path means that the economy is no longer in its Solow growth model long-run equilibrium.

Box 4.4.2: The Labor-Force Growth Rate Matters

The average country with a labor-force growth rate of less than 1 percent per year has an output-per-worker level that is nearly 60 percent of the U.S. level. The average country with a labor-force growth rate of more than 3 percent per year has an output-per-worker level that is only 20 percent of the U.S. level.

To some degree poor countries have fast labor-force growth rates because they are poor: Causation runs both ways. Nevertheless, high labor-force growth rates are a powerful cause of low capital intensity and relative poverty in the world today.

Figure 4.4.1: The Labor Force Growth Rate Matters



Source: Authors' calculations from Alan Heston, Robert Summers, and Bettina Aten, Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP), October 2002, www.nber.org.

How important is all this in the real world? Does a high rate of labor-force growth play a role in making countries relatively poor not just in economists' models but in reality? It turns out that it is important. Of the 22 countries in the world in 2000 with output-per-worker levels at least half of the U.S. level, 18 had labor-force growth rates of less than 2 percent per year, and 12 had labor-force growth rates of less than 1 percent per year. The additional investment requirements imposed by rapid labor-force growth are a powerful reducer of capital intensity and a powerful obstacle to rapid economic growth.

It takes time, decades and generations, for the economy to converge to its new balanced-growth path equilibrium, and thus for the shift in labor-force growth to affect average prosperity and living standards. But the time needed is reason for governments that value their countries' long-run prosperity to take steps now (or even sooner) to start assisting the demographic transition to low levels of population growth. Female education, social changes that provide women with more opportunities than being a housewife, inexpensive birth control—all these pay large long-run dividends as far as national prosperity levels are concerned.

U.S. President John F Kennedy used to tell a story of a retired French general, Marshal Lyautey, "who once asked his gardener to plant a tree. The gardener objected that the tree was slow-growing and would not reach maturity for a hun dred years. The Marshal replied, 'In that case, there is no time to lose, plant it this afternoon.'"

4.4.2.1 The Algebra of a Higher Labor Force Growth Rate

But rather than calculating example by example, set of parameter values by set of parameter values, we can gain some insight by resorting to algebra, and consider in generality the effect on capital-output ratios and output per worker levels of an increase Δn in the labor force growth rate, following an old math convention of using "Δ" to stand for a sudden and discrete change.

Assume the economy has its Solow growth parameters, and its initial balanced-growth path capital-output ratio

(4.4.4) $$\frac{K}{Y}^*_{in} = \frac{s}{n+g+\delta}$$

with "in" standing for "initial".

And now let us consider an alternative scenario, with "alt" standing for "alternative", in which things had been different for a long time:

(4.4.5) $$\frac{K}{Y}^*_{alt} = \frac{s}{n+\Delta n+g+\delta}$$

(4.4.6) $$\frac{K}{Y}^*_{new} = \left(\frac{s}{n+g+\delta}\right)\left(\frac{n+g+\delta}{(n+g+\delta)+\Delta n}\right)$$

But the first term on the right hand side is just the initial capital-output ratio:

(4.4.7) $$\frac{K}{Y}^*_{new} = \left(\frac{K}{Y}\right)_{in}\left(\frac{1}{1+\frac{\Delta n}{n+g+\delta}}\right)$$

And we know that $\frac{1}{1+x}$ is approximately $1-x$ for small values of x, so we can make an approximation:

(4.4.8) $$\frac{K}{Y}^*_{new} \approx \frac{K}{Y}^*_{old} = -\left(\frac{K}{Y}^*_{old}\right)\left(\frac{\Delta n}{n+g+\delta}\right)$$

Take the proportional change in the denominator of the expression for the balanced-growth capital output ratio. Multiply that proportional change by the initial balanced-growth capital-output ratio. That is the differential we are looking for. And by amplifying or damping that change by α/(1−α) we get the differential for output per worker.

4.4.3 A Shift in the Growth Rate of the Efficiency of Labor

4.4.3.1 Efficiency of Labor the Master Key to Long Run Growth

By far the most important impact on an economy's balanced-growth path values of output per worker, however, is from shifts in the growth rate of the efhciency of labor g. We already know that growth in the efhciency of labor is absolutely essential for sustained growth in output per worker and that changes in g are the only things that cause permanent changes in growth rates that cumulate indehnitely.

Recall yet one more time the capital-output ratio form of the production function:

(4.4.1) $$\left(\frac{Y}{L}\right) = \left(\frac{K}{Y}\right)^{\left(\frac{\alpha}{1-\alpha}\right)}(E)$$

Consider what tells us. We know that a Solow growth model economy converges to a balanced-growth path. We know that the capital-output ratio K/Y is constant along the balanced-growth path. We know that the returns-to-investment parameter α is constant. And so the balanced-growth path level of output per worker Y/L grows only if and only as fast as the efficiency of labor E grows.

Yet when we took a look at the math of an economy on its balanced growth path:

(4.4.2) $$\left(\frac{Y}{L}\right)^* = \left(\frac{s}{n+g+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)}(E)$$

we seem to see that an increase in g raises the denominator of the first term on the right hand side—and so pushes the balanced-growth capital output ratio down and the balanced-growth path level of output per worker associated with any level of the efficiency of labor down as well.

It is indeed the case that—just as in the case of an increased labor force growth rate n—an increased efficiency-of-labor growth rate g reduces the economy's balanced-growth capital-output ratio s/(n + g - δ). Why? Because, analogously with an increase in the labor force, increases in the efficiency of labor allow each worker to do the work of more, but they need the machines and buildings to do them. The faster the rate of growth of the efficiency of la or, the larger the share of current investment that must go to keep up with the rising efficiency of old members of the labor force and supply them with the capital they need to be productive. Thus the lower will be the amount of investment that can be devoted to building up or maintaining the average ratio of capital to output.

4.4.3.2 The Algebra of Shifting the Efficiency-of-Labor Growth Rate

The arithmetic and algebra are, for the beginning and the middle, the same as they were for an increase in the rate of labor force growth:

Assume the economy has its Solow growth parameters, and its initial balanced-growth path capital-output ratio

(4.4.4) $$\frac{K}{Y}^*_{in} = \frac{s}{n+g+\delta}$$

with "in" standing for "initial".

And now let us consider an alternative scenario, with "alt" standing for "alternative", in which things had been different for a long time, with a higher efficiency-of-labor growth rate g+Δg since some time t=0 now far in the past:

(4.4.5) $$\frac{K}{Y}^*_{alt} = \frac{s}{n+g+\Delta g+\delta}$$

(4.4.6) $$\frac{K}{Y}^*_{new} = \left(\frac{s}{n+g+\delta}\right)\left(\frac{n+g+\delta}{(n+g+\delta)+\Delta g}\right)$$

But the first term on the right hand side is just the initial capital-output ratio:

(4.4.7) $$\frac{K}{Y}^*_{new} = \left(\frac{K}{Y}\right)_{in}\left(\frac{1}{1+\frac{\Delta g}{n+g+\delta}}\right)$$

And we know that $\frac{1}{1+x}$ is approximately $1 - x$ for small values of x, so we can make an approximation:

(4.4.8) $$\frac{K}{Y}^*_{new} \approx \frac{K}{Y}^*_{old} = -\left(\frac{K}{Y}^*_{old}\right)\left(\frac{\Delta gn}{n+g+\delta}\right)$$

Take the proportional change in the denominator of the expression for the balanced-growth capital output ratio. Multiply that proportional change by the initial balanced-growth capital-output ratio. That is the differential in the balanced-growth capital-output ratio that we are looking for.

But how do we translate that into a differential for output per worker? In the case of an increase in the labor force growth rate, it was simply by amplifying or damping the change in the balanced-growth capital-output ratio by α/(1−α) in order to get the differential for output per worker. We could do that because the efficiency-of-labor at every time t E_t was the same in both the initial and the alternative scenarios.

That is not the case here.

Here, the efficiency of labor was the same in the initial and alternative scenarios back at time 0, now long ago. Since then E has been growing at its rate g in the initial scenario, and at its rate $g + \Delta g$ in the alternative scenario, and so the time subscripts will be important, thus:

(4.4.8) $$\left(\left(\frac{Y_t}{L_t} \right)^* \right)_{in} = \left(\frac{s}{n+g+\delta} \right)^{\alpha/(1-\alpha)} (1+g)^t E_0$$

(4.4.9) $$\left(\left(\frac{Y_t}{L_t} \right)^* \right)_{alt} = \left(\frac{s}{(n+g+\delta)+\Delta g} \right)^{\alpha/(1-\alpha)} (1+g+\Delta g)^t E_0$$

Now divide (4.4.9) by (4.4.8) to get the ratio of output per worker under the alternative and initial scenarios:

(4.4.10) $$\left(\frac{(Y_t/L_t)_{alt}}{(Y_t/L_t)_{in}} \right)^* = \left(\frac{n+g+\delta}{(n+g+\delta)+\Delta g} \right)^{\alpha/(1-\alpha)} \left(\frac{(1+g+\Delta g)^t E_0}{(1+g)^t E_0} \right)$$

which, crossing our fingers and hoping that approximations are good enough, is almost:

(4.4.11) $$\left(\frac{(Y_t/L_t)_{alt}}{(Y_t/L_t)_{in}} \right)^* = \left(\frac{n+g+\delta}{(n+g+\delta)+\Delta g} \right)^{\alpha/(1-\alpha)} (1+\Delta g)^t$$

Thus we see that in the long run, as the second term on the right hand side compounds as t grows, balanced-growth path output per worker under the alternative becomes first larger and then immensely larger than output per worker under the initial scenario. Yes, the balanced-growth path capital-output ratio is lower. But the efficiency of labor at any time t is higher, and then vastly higher if $\Delta g t$ has had a chance to mount up and thus $(1 + \Delta g)^t$ has had a chance to compound.

Yes, a positive in the efficiency of labor growth g does reduce the economy's balanced-growth path capital-output ratio. But these effects are overwhelmed by the more direct effect ofa larger g on output per worker. It is the economy with a high rate of efficiency of labor force growth g that becomes by far the richest over time. This is our most important conclusion. In the very longest run, the growth rate of the standard of living—of output per worker—can change if and only if the growth rate of labor efficiency changes. Other factors—a higher saving-investment rate, lower labor-force growth rate, or lower depreciation rate—can and down. But their effects are short and medium effects: They do not permanently change the growth rate of output per worker, because after the economy has converged to its balanced growth path the only determinant of the growth rate of output per worker is the growth rate of labor efficiency: both are equal to g.

Thus, if we are to increase the rate of growth of the standard of living *permanently*, we must pursue policies that increase the rate at which labor efficiency grows—policies that enhance technological and organizational progress, improve worker skills, and add to worker education.

Box 4.4.3: Shifting the Growth Rate of the Efficiency of Labor

What are the effects of an increase in the rate of growth of the efficiency of labor? Let's work through an example:

Suppose we have, at some moment we will label time 0, t=0, an economy on its balanced growth path with a savings rate s of 20% per year, a labor force growth rate n or 1% per year, a depreciation rate δ of 3% per year, an efficiency-of-labor growth rate g of 1% per year, and a production function curvature parameter α of 1/2. Suppose that at that moment t=0 the labor force L_0 is 150 million, and the efficiency of labor E_0 is 35000.

It is straightforward to calculate the economy at that time 0. Because the economy is on its balanced growth path, its capital-output ratio K/Y is equal to the balanced-growth path capital-output ratio (K/Y)*:

$$\frac{K_0}{Y_0} = \left(\frac{K}{Y}\right)^* = \frac{s}{n+g+\delta} = \frac{0.2}{0.01+0.01+0.03} = \frac{0.2}{0.05} = 4$$

And with an efficiency of labor value $E_0 = 70000$, output per worker at time zero is:

$$\frac{Y_0}{L_0} = \left(\frac{K_0}{Y_0}\right)^{\left(\frac{\alpha}{1-\alpha}\right)}(E_0) = (4)^{\left(\frac{0.5}{1-0.5}\right)}(70000) = (4)^1(35000) = 140000$$

Since the economy is on its balanced growth path, the rate of growth of output per worker is equal to the rate of growth of efficiency per worker. Since the efficiency of labor is growing at 1% per year, we can calculate what output per worker would be at any future time t should the parameters describing the economy remain the same:

$$\left(\frac{Y_t}{L_t}\right)_{ini} = \left(\frac{Y_0}{L_0}\right) e^{gt} = (140000)e^{(0.01)t}$$

where the subscript "ini" tells us that this value belongs to an economy that retains its initial parameter values into the future. Thus 69 years into the future, at t=69:

$$\left(\frac{Y_{69}}{L_{69}}\right)_{ini} = (140000)e^{(0.01)t} = (140000)(1.9937) = 279120$$

Now let us consider either a jump in g—a permanent, discontinuous change—or an alternative scenario in which output per worker is the same in year 0 but in which the efficiency of labor growth rate g is a higher rate. Suppose $g_{alt} = g_{ini} + \Delta g$, with the subscript "alt" reminding us that this parameter or variable belongs to the alternative scenario just as "ini" reminds us of the initial scenario or set of values. How do we forecast the growth of the economy in an alternative scenario—in this case, in an alternative scenario in which $\Delta g = 0.02$?

The first thing to do is to calculate the balanced growth path steady-state capital-output ratio in this alternative scenario. Thus we calculate:

$$\left(\frac{K}{Y}\right)^*_{alt} = \frac{s}{n+g+\Delta g+\delta} = \frac{0.2}{0.01+0.03+0.03} = \frac{0.2}{0.07} = 2.857$$

The steady-state balanced growth path capital-output ratio is much lower in the alternative scenario than it was in the initial scenario: 2.857 rather than 4. The capital-output ratio, of course, does not drop instantly to its new steady-state value. It takes time for the transition to occur.

While the transition is occurring, the efficiency of labor in the alternative scenario is growing at not 1% but 3% per year. We can thus calculate the alternative scenario balanced growth path value of output per worker as:

$$\left(\frac{Y_t}{L_t}\right)^*_{alt} = \left[\frac{s}{n+g+\Delta g+\delta}^{\left(\frac{\alpha}{1-\alpha}\right)}\right] e^{(g+\Delta g)t} = \left[(2.857)^1\right] E_0 e^{(0.03)t}$$

And in the 69th year this will be:

$$\left(\frac{Y_{69}}{L_{69}}\right)^*_{alt} = \left[(2.857)^1\right] (35000) e^{(0.03)69} = 792443$$

How good would this balanced growth path value be as an estimate of the actual behavior of the economy? We know that a Solow growth model economy closes a fraction $(1-\alpha)(n+g+\delta)$ of the gap between its current position and its steady-state balanced growth path capital-output ratio each period. For our parameter values $(1-\alpha)(n+g+\delta) = 0.035$. That gives us about 20 years as the period needed to converge halfway to the balanced growth path. 69 years is thus about 3.5 such halvings of the gap—meaning that the economy will close 9/10 of the way. Thus assuming the economy is on its alternative scenario balanced growth path in year 69 is not a bad assumption.

But if we want to calculate the estimate exactly?

$$\left(\frac{K_{69}}{Y_{69}}\right)_{alt} = \left(\frac{s}{n+g+\Delta g+\delta}\right) + \left[\frac{K_0}{Y_0} - \frac{s}{n+g+\Delta g+\delta}\right] e^{-(1-\alpha)(n+g+\Delta d+\delta)(69)}$$

$$\left(\frac{K_{69}}{Y_{69}}\right)_{alt} = \left(\frac{0.20}{0.01+0.01+0.02+0.03}\right) + \left[4 - \frac{0.20}{0.01+0.01+0.02+0.03}\right] e^{-(0.5)(0.01+0.01+0.02+0.03)(69)}$$

$$\left(\frac{K_{69}}{Y_{69}}\right)_{alt} = 2.857 + [4-2.857]\, e^{-(0.035)(69)} = 2.857 + (1.143) e^{-(0.035)(69)} =$$
$$2.857 + (1.143)(0.089) = 2.959$$

And with a year-69 capital-output ratio in the alternative scenario of 2.959, output per worker is then:

$$\left(\frac{Y_{69}}{L_{69}}\right)_{alt} = \left(\frac{K_{69}}{Y_{69}}\right)^{\frac{\alpha}{1-\alpha}} [E_{69alt}] = (2.959) \left[E_0 e^{(g+\Delta g)(69)}\right] = (2.959) \left[(2.959) e^{(0.03)(69)}\right] =$$

$$\left(\frac{Y_{69}}{L_{69}}\right)_{alt} = (2.959)(35000)(7.925) = 820752$$

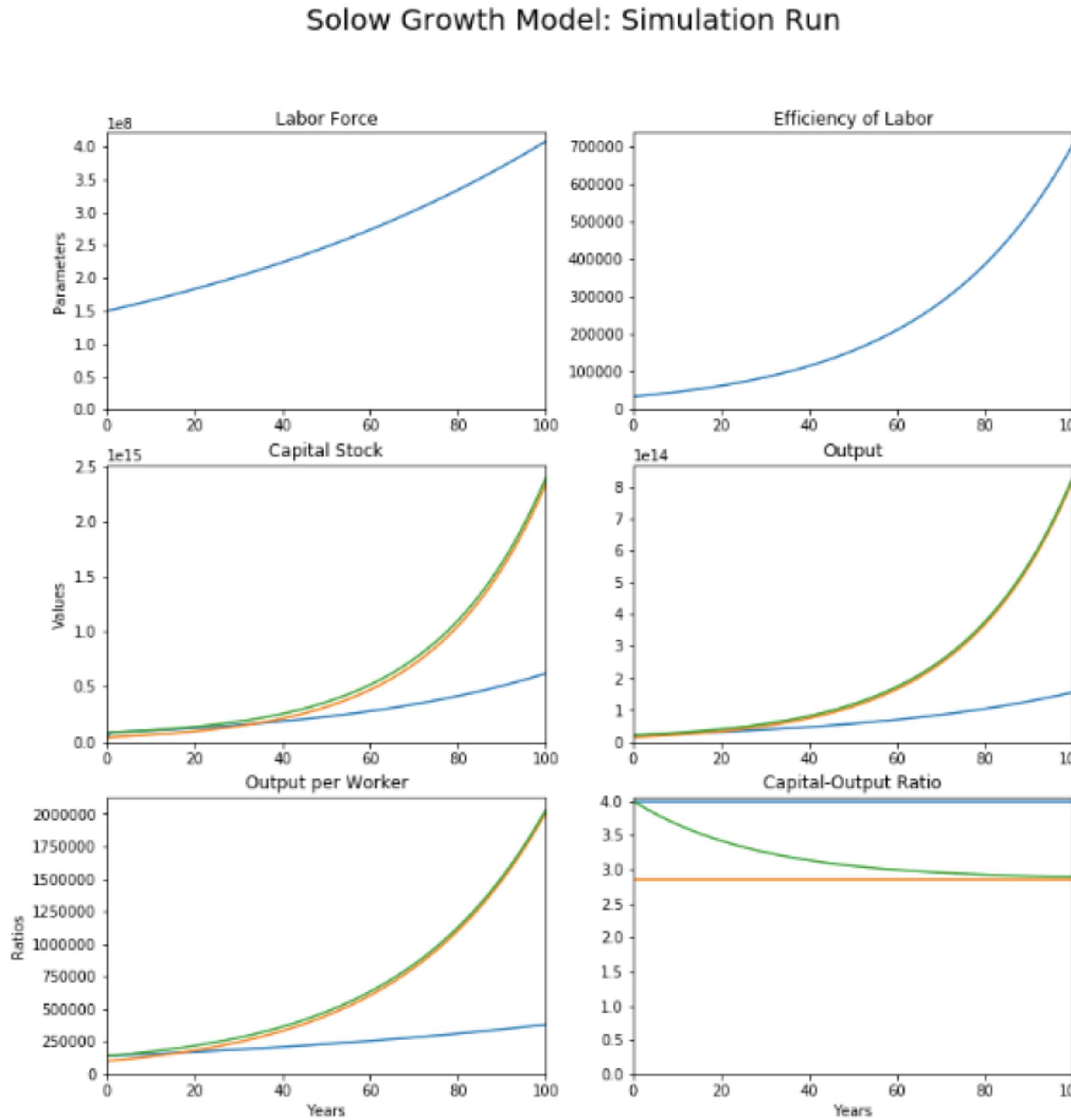
An actual alternative scenario output per worker value of 820752 in year 69; a balanced growth path alternative scenario value of 792443; and an initial parameter values scenario value of 279120; all from a year-0 value of 140000 for output per worker.

The takeaways are three:

1. For these parameter values, 69 years are definitely long enough for you to make the assumption that the economy has converged to its Solow model balanced growth path. One year no. Ten years no. Sixty-nine years, yes.
2. Shifts in the growth rate g of the efficiency of labor do, over time, deliver enormous differentials in output per worker across scenarios.
3. The higher efficiency of labor economy is, in a sense, a less capital intensive economy: only 2.959 years' worth of current production is committed to and tied up in the economy's capital stock in the alternative scenario, while 4 years' worth was tied up in the initial scenario. But the reduction in output per worker generated by a lower capital-output ratio is absolutely swamped by the faster growth of the efficiency of labor, and thus the much greater value of the efficiency of labor in the alternative scenario comes the 69th year.

Desire not an algebraic but a visual, graphical depiction of the difference between the initial and the alternative scenarios for the alternative scenario with a 2% per year faster growth rate of labor efficiency? Continue below to Box 4.4.4

Box 4.4.4: A Faster Efficiency of Labor Growth Rate: Year by Year



The figure panel above shows the initial and alternative balanced growth paths for an economy on the initial growth path until year zero, and then experiencing a two percentage point per year permanent jump in its efficiency of labor growth rate. The blue curves are the initial scenario balanced growth path values; the orange curves are the alternative scenario values; the green curves are the track of the variables in the economy as it converges from the initial to the alternative balanced growth path.

The figure panel above has definite values for the parameters. There is a $\Delta g = 0.02$—a two percentage point permanent jump in the yearly proportional growth rate of the efficiency of labor g. There are no changes to the labor force growth rate n, the savings rate s, or any other values of parameters or initial conditions. They are:

- savings rate s is 20%
- labor force growth rate n is 1% per year
- depreciation rate δ is 3% per year
- capital share production function parameter α is 0.5
- initial efficiency of labor E_0 is 35000
- the initial scenario rate of growth of the efficiency of labor g is 1% per year.

But you do not have to restrict yourself to this one figure panel and these parameter values. The code cell below contains oru function for calculating and graphing the levels of Solow growth model variables in simulations—the simulated track of the economy, plus both the initial and the alternative scenario balanced growth paths, with initial and alternative scenarios distinguished by values of Δn, the change in the labor force growth rate, Δg, the change in the labor efficiency growth rate, and Δs, the change in the savings investment share.

After running the code cell below, create a new code cell and in it call the function:

```
sgm_bgp_100yr_run(L0, E0, n=0.01, g=0.01, s=0.20,
    alpha=0.5, delta=0.03, Delta_s=0, Delta_g=0, Delta_n=0,
    T = 100)
```

The first two arguments are required, and are the initial time zero values for the labor force and the efficiency of labor. The other values are optional, and if omitted will be set to their default values in the function definition.

As you simulate different possibilities particular attention to how differences across simulations in the capital share parameter α and the depreciation rate δ cause a Solow growth model economy to react differently to shifts in labor force growth, labor efficiency growth, and the savings-investment share caused by changes or differences in economic policy and the economic environment.

4.4.4 Shifts in the Saving Rate s

4.4.4.1 The Most Common Policy and Environment Shock

Shifts in labor force growth rates do happen: changes in immigration policy, the coming of cheap and easy contraception (or, earlier, widespread female literacy), or increased prosperity and expected prosperity that trigger "baby booms" can all have powerful and persistent effects on labor force growth down the pike. Shifts in the growth of labor efficiency growth happen as well: economic policy disasters and triumphs, countless forecasted "new economies" and "secular stagnations", and the huge economic shocks that were the first and second Industrial Revolutions—the latter inaugurating that global era of previously unimagined increasing prosperity we call modern economic growth—push an economy's labor efficiency growth rate g up or down and keep it there.

Nevertheless, the most frequent sources of shifts in the parameters of the Solow growth model are shifts in the economy's saving-investment rate. The rise of politicians eager to promise goodies—whether new spending programs or tax cuts — to voters induces large government budget deficits, which can be a persistent drag on an economy's saving rate and its rate of capital accumulation. Foreigners become alternately overoptimistic and overpessimistic about the value of investing in our country, and so either foreign saving adds to or foreign capital flight reduces our own saving- investment rate. Changes in households' fears of future economic disaster, in households' access to credit, or in any of numerous other factors change the share of household income that is saved and invested. Changes in government tax policy may push after-tax returns up enough to call forth additional savings, or down enough to make savings seem next to pointless. Plus rational or irrational changes in optimism or pessimism—what John Maynard Keynes labelled the "animal spirits" of individual entrepreneurs, individual financiers, or bureaucratic committees in firms or banks or funds all can and do push an economy's savings-investment rate up and down.

4.4.4.2 Analyzing a Shift in the Saving Rate s

What effects do changes in saving rates have on the balanced-growth path levels of Y/L?

The higher the share of national product devoted to saving and gross investment—the higher is s—the higher will be the economy's balanced-growth capital-output ratio s/(n + g + δ). Why? Because more investment increases the amount of new capital that can be devoted to building up the average ratio of cap ital to output. Double the share of national product spent on gross investment, and you will find that you have doubled the economy's capital intensity, or its average ratio of capital to output.

As before, the equilibrium will be that point at which the economy's savings effort and its investment requirements are in balance so that the capital stock and output grow at the same rate, and so the capital-output ratio is constant. The savings effort of society is simply sY, the amount of total output devoted to saving and investment. The investment requirements are the amount of new capital needed to replace depreciated and worn-out machines and buildings, plus the amount needed to equip new workers who increase the labor force, plus the amount needed to keep the stock of tools and machines at the disposal of more efficient workers increasing at the same rate as the efficiency of their labor.

(4.4.12) $$sY = (n + g + \delta)K$$

And so an increase in the savings rate s will, holding output Y constant, call forth a proportional increase in the capital stock at which savings effort and investment requirements are in balance: increase the saving-investment rate, and you double the balanced-growth path capital-output ratio:

(4.4.4) $$\frac{K}{Y}^*_{in} = \frac{s}{n+g+\delta}$$

(4.4.13) $$\frac{K}{Y}^*_{alt} = \frac{s+\Delta s}{n+g+\delta}$$

(4.4.14) $$\frac{K}{Y}^*_{alt} - \frac{K}{Y}^*_{in} = \frac{\Delta s}{n+g+\delta}$$

with, once again, balanced growth path output per worker amplified or damped by the dependence of output per worker on the capital-output ratio:

(4.4.2) $$\left(\frac{Y}{L}\right)^* = \left(\frac{s}{n+g+\delta}\right)^{\left(\frac{\alpha}{1-\alpha}\right)}(E)$$

Table 4.4.1: Effects of Increases In Parameters on the Solow Growth Model

When there is an increase in the parameter . . .	The effect on . . . Equilibrium K/Y	Level of Y	Level of Y/L	Permanent Growth Rate of Y	Permanent Growth Rate of Y/L
s saving-investment rate	Increases	Increases	Increases	No change	No change
n labor-force growth rate	Decreases	Increases	Decreases	Increases	No change
δ depreciation rate	Decreases	Decreases	Decreases	No change	No change
g efficiency of labor growth rate	Decreases	Increases	Increases	Increases	Increases

Box 4.4.5: An Increase in the Saving-Investment Rate: An Example

To see how an increase in the economy's saving rate s changes the balanced-growth path for output per worker, consider an economy in which the parameter α is 2/3, the rate of labor-force growth n is 1 percent per year, the rate of labor efficiency growth g is 1.5 percent per year, and the depreciation rate δ is 3.5 percent per year.

Suppose that the saving rate s, which was 18 percent, suddenly and permanently jumped to 24 percent of output.

Before the increase in the saving rate, when s was 18 percent, the balanced-growth equilibrium capital-output ratio was:

$$\left(\frac{K}{Y}\right)^*_{ini} = \frac{s}{n+g+\delta} = \frac{0.18}{0.01+0.015+0.035} = 3$$

After the increase in the saving rate, the new balanced-growth equilibrium capital- output ratio will be:

$$\left(\frac{K}{Y}\right)^*_{alt} = \frac{s+\Delta s}{n+g+\delta} = \frac{0.24}{0.01+0.015+0.035} = 4$$

Before the increase in saving, the balanced-growth path for output per worker was:

$$\left(\frac{Y}{L}\right)^*_{ini} = \left(\frac{K}{Y}\right)^{*\left(\frac{\alpha}{1-\alpha}\right)}_{ini}(E) = (3)^{\left(\frac{2/3}{1-2/3}\right)}(E) = 3^2(E) = 9E$$

After the increase in saving, the balanced-growth path for output per worker would be:

$$\left(\frac{Y}{L}\right)^*_{alt} = \left(\frac{K}{Y}\right)^{*\left(\frac{\alpha}{1-\alpha}\right)}_{alt}(E) = (4)^{\left(\frac{2/3}{1-2/3}\right)}(E) = 4^2(E) = 16E$$

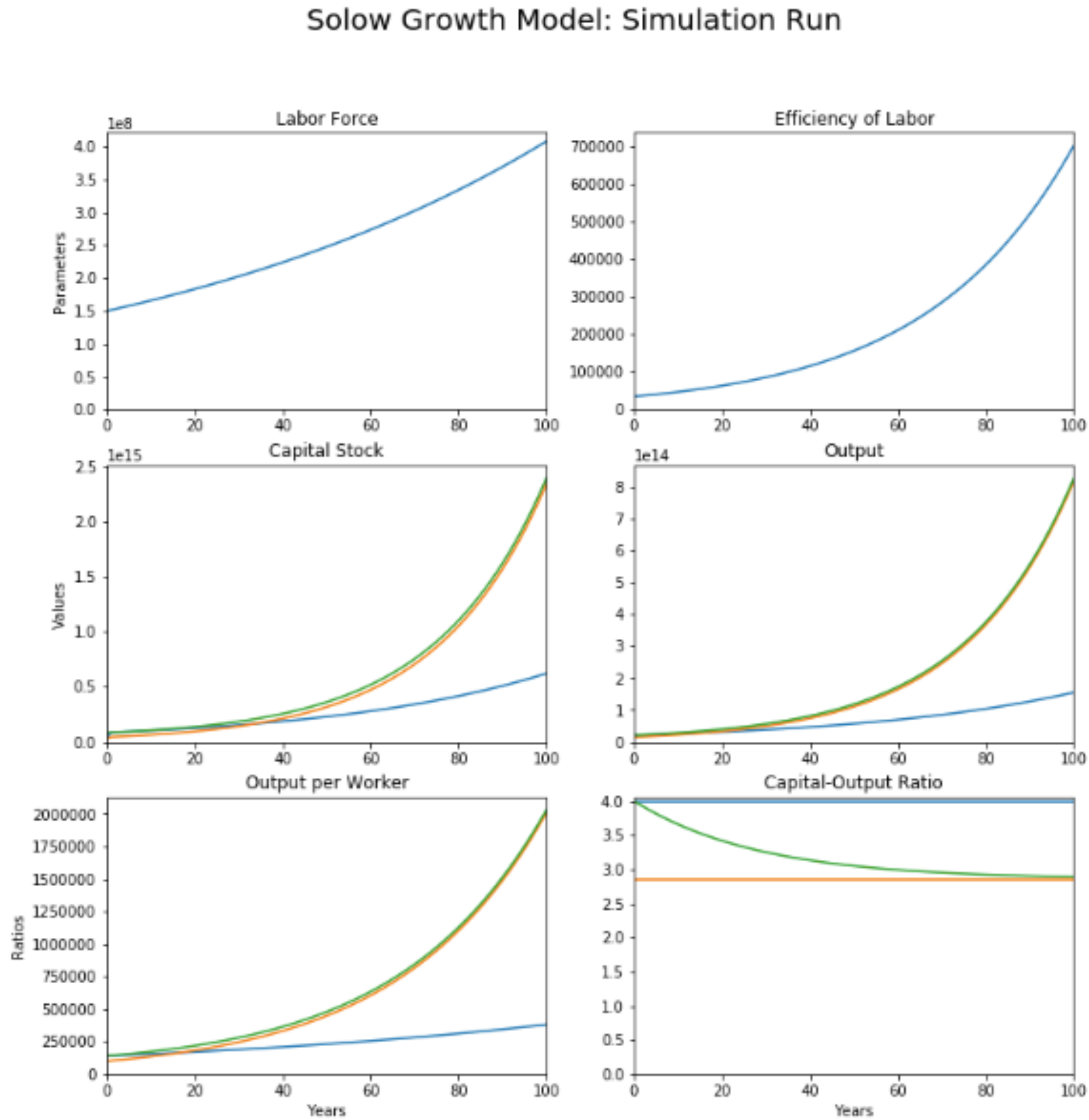
Divide the second equation by the first. We see that balanced-growth path output per worker after the jump in the saving rate is higher by a factor of 16/9, or fully 78 percent higher.

Just after the increase in saving has taken place, the economy is still on its old, balanced-growth path. But as decades and generations pass the economy converges to its new balanced-growth path, where output per worker is not 9 but 16 times the efficiency of labor. The jump in capital intensity makes an enormous differ ence for the economy's relative prosperity.

Note that this example has been constructed to make the effects of capital inten sity on relative prosperity large: The high value for the diminishing-returns-to- investment parameter a means that differences in capital intensity have large and powerful effects on output-per-worker levels.

But even here, the shift in saving and investment does not permanently raise the economy's growth rate. After the economy has settled onto its new balanced-growth path, the growth rate of output per worker returns to the same 1.5 percent per year that is g, the growth rate of the effciency of labor.

The figure immediately below charts the effects of such a permanent jump in the savings-investment share s with such a high production function α parameter:



And the code cell below allows you to run your own simulations:

Box 4.4.6: Estimating the Effects of Policy Changes: An Example

In late 2017 and early 2018 the Trump administration and the Republican congressional caucuses pushed through a combined tax cut and a relaxation of spending caps to the tune of increasing the federal government budget deficit by about 1.4% of GDP. These policy changes were intended to be permanent.

Not the consensus but the center-of-gravity analysis by informed opinion in the economics profession of the effects on long-run growth of such a *permanent* change in fiscal policy would have made the following points:

1. The U.S. economy at the start of 2018 was roughly at full employment, or at least the Federal Reserve believed that it was at full employment and was taking active steps to keep spending from rising faster than their estimate of the trend growth of the economy, so a long-run Solow growth model analysis would be appropriate.
2. The economy's savings-investment effort rate, s, has two parts: private and government saving: $s = s_p + s_g$.
3. The private savings rate s_p is very hard to move by changes in economic policy. Policy changes that raise rates of return on capital—interest and profit rates—both make it more profitable to save and invest more but also make us richer in the future, and so diminish the need to save and invest more. These two roughly offset.
4. Therefore, when the economy is at full employment, changes in overall savings are driven by changes in the government contribution: $\Delta s = \Delta s_g$.
5. And an increase in the deficit is a reduction in the government savings rate.

The standard center-of-gravity analysis would thus start by assuming that the economy was on its balanced growth path, and investigate the consequences of a reduction in s by 1.4% points in order to get an estimate of the effect of this policy shift if it were to be a permanent change.

Set up the Solow growth model, with the Labor force growth rate n = 1.0% per year, the labor efficiency growth rate g = 1.5% per year, the depreciation rate δ = 3% per year, the production function diminishing returns to investment parameter α = 1/3, and the initial efficiency of labor E_0 = 65000. That produces an initial state of the economy's balanced growth path of:

$$\left(\frac{K}{Y}\right)^*_{ini} = 4$$

$$\left(\frac{Y}{L}\right)^*_{ini} = \left(\frac{K}{Y}\right)^{*\left(\frac{\alpha}{1-\alpha}\right)}_{ini} E_0 = 4^{(1/2)}(65000) = 130000$$

Along the alternative balanced growth path, the same variables are:

$$\left(\frac{K}{Y}\right)^*_{alt} = \left(\frac{0.22-0.014}{0.01+0.015+0.03}\right)^{\left(\frac{1/3}{1-1/3}\right)} = 3.745$$

$$\left(\frac{Y}{L}\right)^*_{alt} = \left(\frac{K}{Y}\right)^{*\left(\frac{\alpha}{1-\alpha}\right)}_{alt} E_0 = \left(\frac{0.22-0.014}{0.01+0.015+0.03}\right)^{\left(\frac{1/3}{1-1/3}\right)}(65000) = 3.745^{(1/2)}(65000) = 130000$$

That is, the alternative balanced growth path has an output per worker level 3.3 percent below the initial path The policy is expensive for the economy in the long run.

How fast does this growth retardation make itself felt? We know that the velocity of convergence v_c in the Solow growth model is:

$$v_c = -(1 - \alpha)(n + g + \delta)$$

In this case:

$$v_c = -(1 - \alpha)(n + g + \delta) = -(1 - 1/3)(0.01 + 0.015 + 0.035) = -0.0329$$

The economy closes about 1/30 of the gap between its initial and its alternative balanced growth path every year. The first-year effect is thus about (-0.033)(0.33) = -0.001: a drop in the growth rate of 0.1% point, and a drop in the level of 0.1% point after one year. After 10 years, the economy will have closed about 28 percent of the 3.3 percentage point gap—a total effect on the level of real GDP ten years out of 0.9%: nine-tenths of a percentage point.

Box 4.4.7: Speed of Convergence and Estimating the Effects of Policy Changes: An Alternative

It is worth noting an alternative calculation of the likely effects of the Trump administration's economic policies, carried out by four Stanford economists and five others. The most important thing to know to understand and evaluate this calculation is that all nine of these economists are strong Republicans. They wrote http://delong.typepad.com/2017-11-26-nine-unprofessional-republican-economists.pdf (http://delong.typepad.com/2017-11-26-nine-unprofessional-republican-economists.pdf), in a piece that was notionally a letter to U.S. Treasury Secretary Steven Mnuchin but that was in actuality primarily intended to be published in the *Wall Street Journal* to influence the debate, that Trump administration fiscal policy—the tax cut—would:

> increase... the capital stock... raise the level of GDP in the long run by just over 4%. If achieved over a decade, the associated increase in the annual rate of GDP growth would be about 0.4% per year.... [In] the House and Senate bills... the increase in capital accumulation would be less, and the gain in the long-run level of GDP would be just over 3%, or 0.3% per year for a decade...

The four Stanford University economists are:

- **Michael J. Boskin**, Tully M. Friedman Professor of Economics, Stanford University; Chairman of the Council of Economic Advisers under President George H.W. Bush
- **John Cogan**, Leonard and Shirley Ely Senior Fellow, Hoover Institution, Stanford University; Deputy Director of the Office of Management and Budget under President Ronald Reagan
- **George P. Shultz**, Thomas W. and Susan B. Ford Distinguished Fellow, Hoover Institution, Stanford University; Secretary of State under President Ronald Reagan; Secretary of the Treasury under President Richard Nixon

- **John. B. Taylor**, Mary and Robert Raymond Professor of Economics, Stanford University; Undersecretary of the Treasury for International Affairs under President George W. Bush

The five others are:

- **Robert J. Barro**, Paul M. Warburg Professor of Economics, Harvard University
- **Douglas Holtz-Eakin**, President, American Action Forum, former director of the Congressional Budget Office
- **Glenn Hubbard**, Dean and Russell L. Carson Professor of Finance and Economics (Graduate School of Business) and Professor of Economics (Arts and Sciences), Columbia University; Chairman of the Council of Economic Advisers under President George W. Bush
- **Lawrence B. Lindsey**, President and Chief Executive Officer, The Lindsey Group; Director of the National Economic Council under President George W. Bush
- **Harvey S. Rosen**, John L. Weinberg Professor of Economics and Business Policy, Princeton University; Chairman of the Council of Economic Advisers under President George W. Bush

Their conclusions—"the gain in the long-run level of GDP would be just over 3%, or 0.3% per year for a decade..."—look in their effects on levels of output per worker like the calculation in box 4.4.6, with one crucial difference: the sign is reversed. In 4.4.6, the first order effect of the policy changes was to reduce national savings and investment and thus make America a less capital intensive and poorer economy. And this calculation, the first order effect is to raise national savings and investment and us make America a more capital intensive and richer economy. Moreover, the effect on the growth rate is not only of the wrong sign, but three times the magnitude: instead of a slowdown in annual growth of 0.1% point, there is a speedup of

Why the difference?

Why does not the increased government deficit and thus government anti-saving reduce the national savings investment rate s? The authors do not say.

Where is the analysis stating that increased after tax rates of return on savings and investment have offsetting substitution and income effects, with the substitution effect raising saving and the income effect lowering it? That analysis, also, is absent.

What, then, is present? This:

> Fundamental tax reform... [is] a set of tax changes that reduces tax distortions on productive activities (for example, business investment and work) and broadens the tax base to reduce tax differences among similarly situated businesses and individuals. Fundamental tax reform should also advance the objectives of fairness and simplification.... The proposals emerging from the House, Senate, and President Trump's administration, fall squarely within this tradition.... There is some uncertainty about just how much additional investment is induced by reductions in the cost of capital, but... many economists believe that a 10% reduction in the cost of capital would lead to a 10% increase in the amount of investment. Simultaneously reducing the corporate tax rate to 20% and moving to immediate expensing of equipment and intangible investment would reduce the user cost by an average of 15%, which would increase the demand for capital by 15%.... Such an increase in the capital stock would raise the level of GDP... just over 3%, or 0.3% per year for a decade...

That's all she writes. And note: "many" economists—not "most economists", not "nearly all economists", not "the center of gravity of informed economic opinion".

And the claims about "the proposals emerging from the House, Senate, and President Trump's administration" being "within this tradition" of "broaden[ing] the tax base to reduce tax differences among similarly situated businesses and individuals... advanc[ing] the objectives of fairness and simplification..." are simply false.

Even more alarming than the reversal-of-sign of the effect, is the estimate of the growth rate: a jump of + 0.3 percentage points per year. It comes from the nine economists' observation that:

> increase... [would] raise the level of GDP in the long run by just over 4%. If achieved over a decade, the associated increase in the annual rate of GDP growth would be about 0.4% per year.... [In] the House and Senate bills... the increase in capital accumulation would be less, and the gain in the long-run level of GDP would be just over 3%, or 0.3% per year for a decade...

But the nine economists know just as well as you do that only 28 percent of the total gain accrues in the first decades, not all of it.

When challenged by former U.S. Treasury Secretary Lawrence Summers and former Council of Economic Advisers Chair Jason Furman https://www.washingtonpost.com/news/wonk/wp/2017/11/28/lawrence-summers-dear-colleagues-please-explain-your-letter-to-steven-mnuchin/?utm_term=.9d690352f4b3 (https://www.washingtonpost.com/news/wonk/wp/2017/11/28/lawrence-summers-dear-colleagues-please-explain-your-letter-to-steven-mnuchin/?utm_term=.9d690352f4b3):

> Since you are explicitly talking about 10-year growth rates in your letter, would it not be better to... show that the effect in the 10th year is less than one-third of the long-run effect, translating into an annual growth rate of less than 0.1 percentage point?...

The nine economists denied that they had made claims about the speed of adjustment to the post policy change blaanced growth path and so offered a prediction that real GDP growth would be boosted by not 0.1% (or -0.1%) but rather 0.3% points per year over the next decade https://www.washingtonpost.com/news/wonk/wp/2017/11/29/economists-respond-to-summers-furman-over-mnuchin-letter/?utm_term=.8d4d8991717a (https://www.washingtonpost.com/news/wonk/wp/2017/11/29/economists-respond-to-summers-furman-over-mnuchin-letter/?utm_term=.8d4d8991717a):

> **First point you raised**: Our letter addresses the impact of corporate tax reform on GDP; we did not offer claims about the speed of adjustment to a long-run result...

We believe that Stanford (and Harvard, and Columbia, and Princeton, and the American Action Forum, and the Lindsey Group) have a serious problem here: As Berkeley medieval history professor Ernst Kantorowicz wrote http://www.lib.berkeley.edu/uchistory/archives_exhibits/loyaltyoath/symposium/kantorowicz.html (http://www.lib.berkeley.edu/uchistory/archives_exhibits/loyaltyoath/symposium/kantorowicz.html) back in the 1940s, shortly before being fired for refusing to take a loyalty oath demanded by the Regents of the University of California, academic freedom is a grave and serious thing:

> Professions... entitled to wear a gown: the judge, the priest, the scholar. This garment stands for its bearer's maturity of mind, his independence of judgment, and his direct responsibility to his conscience and to his God.... They should be the very last to allow themselves to act under duress and yield to pressure. It is... shameful and undignified... an affront and a violation of both human sovereignty and professional dignity... to bully... under... economic coercion... compell[ing] either giv[ing] up... tenure or... his freedom of judgment, his human dignity and his responsible sovereignty as a scholar...

Those possessing academic freedom are given great latitude so that they can speak what they, after great and considered research and reflection, believe sincerely to be the truth. But this freedom to be responsible solely to one's conscience and God requires that one be responsible to one's conscience and God. But what if bearers of academic freedom fear not God nor their own consciences? What then?

One possibility is to inquire and point out that something has gone wrong, as Summers and Furman did, politely, with:

> Since you are explicitly talking about 10-year growth rates in your letter, would it not be better to... show that the effect in the 10th year is less than one-third of the long-run effect, translating into an annual growth rate of less than 0.1 percentage point?...

inviting the response: "yes, it would have been better; we have made an error; we will correct it".

But that is not the reply Summers and Furman got.

A second possibility is to teach young people the basics of macroeconomics. I hope everybody who read the nine economists letter who had ever taken a macroeconomics course read that "the gain in the long-run level of GDP would be just over 3%, or 0.3% per year for a decade..." and immediately thought: "that is not how the effects of an increase in the economy's capital intensity from a higher savings-investement effort work—these authors, prestigious as their academic appointments may be, are not doing economic analysis but rather playing political Three-Card Monte". I hope everybody who reads this textbook remembers enough of it that they are able to do the work of reading with a jaundiced eye that is clearly needed here.

There was, I should say, a further oblique reply by one of the four Stanford economists, Michael Boskin https://www.project-syndicate.org/commentary/republican-tax-plan-growth-effects-by-michael-boskin-2017-12 (https://www.project-syndicate.org/commentary/republican-tax-plan-growth-effects-by-michael-boskin-2017-12). In it he made points:

1. "Robert Barro..." published a deeper elaboration of the tax plan’s growth effects..." (which saiclaimedd that the long-run balanced growth path boost to the level of output per worker would be not 3 percent but 7 percent).
2. "The current tax bill could... have been better.... But such a bill would not pass Congress."
3. "The question is whether a viable final bill will be better than the status quo."
4. "Barro and I have clearly come to a different conclusion.... While I certainly respect Summers and Furman’s right to their views, I am not about to cede my professional judgment to others, in or out of government."
5. "There are legitimate differences of opinion on how much and how quickly the tax plan will affect investment decisions."
6. "Summers... and DeLong... have made the strongest case I know that equipment investment can have a large impact... much larger than in the conventional models."
7. "I believe that the current reform may well have deviated further from the ideal had we not offered our analysis and advice.... Many factors other than economists’ textbook policy proposals affect the final product."

8. "The actual tax provisions people and businesses will be required to use have yet to be written, and will be determined partly by technical interpretations and regulations in the coming months."

Point (6) seems to me to be a red herring, at least as far as the policy change's effects on the growth rate are concerned. We—DeLong and Summers—believe that α is higher than the 1/3 assumed in the center-of-gravity of informed economic opinion analyses. A higher value of α both stretches out the time it takes for the economy to converge and magnifies the ultimate differential, and these effects roughly cancel out, leaving the near term growth rate effect unchanged. And a higher α magnifies both the boost from higher savings and the drag from those savings being diverted to finance larger government deficits.

The overwhelming impression I get from Boskin's piece is one of extraordinary cognitive dissonance. If I sincerely believed that a policy change was likely to boost America's productivity and wealth by 7 percent, I would not be apologizing for it. I would be crowing from the rooftops. I would not be agreeing that "the current tax bill could... have been better". I would not be saying that the bar is the very low "better than the status quo". I would not be defending my participation in the process on the grounds that the bill would have been worse if I had washed my hands of it. I would not be saying that we need to work hard now to improve it because "the actual tax provisions people and businesses will be required to use have yet to be written". I would not be saying that there are legitimate differences of opinion and that I respect the judgemtns of those who think differently.

I thus read Boskin's piece as, in large part, and perhaps not completely of his intention, a *sotto voce* argument that:

1. We nine economists said in public what we needed to say so that we could get into the room where the decisions were really being made.
2. We nine economists made the bill better than it would have been otherwise.
3. We nine economists will continue to make the implementation of the bill better.

4.4.4.3 Saving and Investment: Prices and Quantities

The same consequences as a low saving rate—a lower balanced-growth capital- output ratio —would follow from a country that makes the purchase of capital goods expensive. An abnormally high price of capital goods can translate a reasonably high saving effort into a remarkably low outcome in terms of actual gross additions to the real capital stock. The late economist Carlos Diaz-Alejandro placed the blame for much of Argentina's poor growth performance since World War II on trade policies that restricted imports and artificially boosted the price of capital goods. Economist Charles Jones reached the same conclusion for India. And economists Peter Klenow and Chang-Tai Hsieh argued that the world structure of prices that makes capital goods relatively expensive in poor countries plays a major role in blocking development.

4.4.4.3 How Important This Is in the Real World

How important is all this in the real world? Does a high rate of saving and investment play a role in making countries relatively rich not just in economists' models but in reality? It turns out that it is important indeed. Of the 22 countries in the world with output-per-worker levels at least half of the U.S. level, 19 have investment that is more than 20 percent of output. The high capital-output ratios generated by high investment efforts are a very powerful source of relative prosperity in the world today.

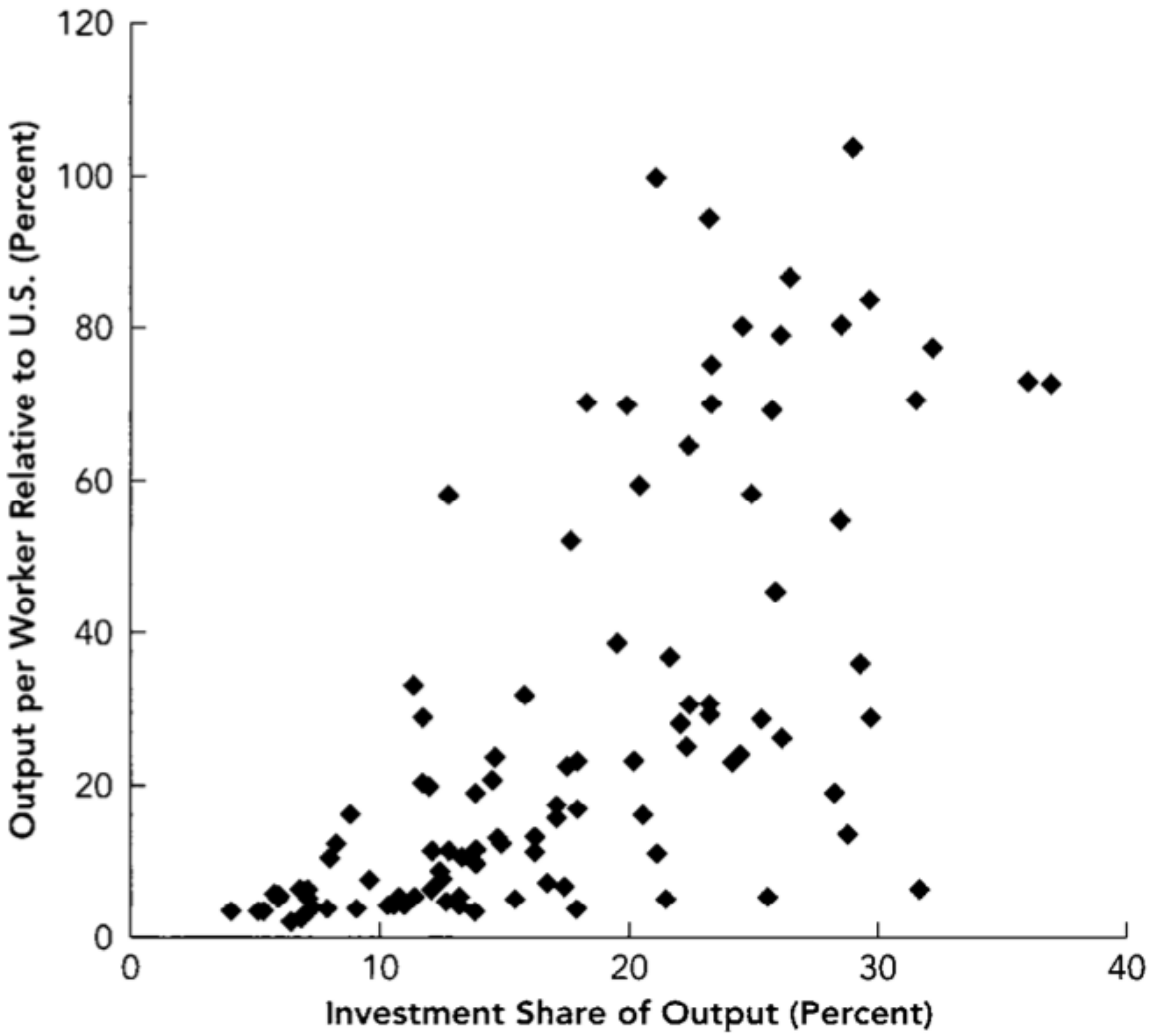
Figure 4.4.2: Savings-Investment Shares of Output and Relative Prosperity

The average country with an investment share of output of more than 25 percent has an output-per-worker level that is more than 70 percent of the U.S. level.

The average country with an investment share of output of less than 15 percent has an output-per-worker level that is less than 15 percent of the U.S. level.

This is not entirely due to a one-way relationship from a high investment effort to a high balanced-growth capital-output ratio: Countries are poor not just because they invest little; to some degree they invest little because they are poor. But much of the relationship is due to investment's effect on prosperity. High saving and investment rates are a very powerful cause of relative wealth in the world today.

Where is the United States on this graph? For these data it has an investment rate of 21 percent of GDP and an output- per-worker level equal (not surprisingly) to 100 percent of the U.S. level.



Source: Authors' calculations from the Penn World Table data constructed by Alan Heston, Robert Summers, and Bettina Aten, www.nber.org.

4.4.5 RECAP: Using the Solow Growth Model

Changes in the economic environment and in economic policy have powerful effects on the economy's long-run economic growth path. In the Solow model we analyze the effects of such changes by looking at their effects on cap ital intensity and on the efficiency of labor. Unless the production function has very little in the way of diminishing returns—unless the parameter α is high—shifts in the growth rate of the efficiency of labor have the most powerful effects on the long-rund estiny in the economy: They change the long-run growth rate of the economy. Shifts in other parameters affect the economy's capital intensity; affect what multiple of the efficiency of labor the balanced-growth path of output per worker follows, and make the economy richer or poorer as it con verges to a new, different balanced-growth path. But only a change in the growth rate of labor efficiency can produce a permanent change in the growth rate of output per worker.

4.4.6 EXERCISES: Using the Solow Growth Model

Task 1: The Effects of a Jump in the Savings Rate:

Do calculations and assign the appropriate values to the variables in the code cells below:

GLOSSARY:

Balanced-growth path: The path toward which total output per worker tends to converge, as the capital-output ratio converges to its equilibrium value.

Capital intensity: The ratio of the capital stock to total potential output, K/Y, which describes the extent to which capital, as opposed to labor, is used to produce goods and services.

Efficiency of labor: The skills and education of the labor force, the ability of the labor force to handle modern technologies, and the efficiency with which the economy's businesses and markets function.

Long-run economic growth: The process by which productivity, living standards, and output increase.

Saving rate: The share of total GDP that an economy saves, s, equal to the sum of household, government, and foreign saving divided by total output.
