Section Exercise for March 30/31: Economic Growth with Answers

1) Suppose that some quantity grows at a steady pace of 0.1% per year. About how long does it take to double? About how long does it take to grow tenfold? A hundredfold?

You can go to, for example, http://wolframalpha.com and start entering expressions like (1 + 0.1%)^10, (1 + 0.1%)^100, and so on until you get (1 + 0.1%)^694 = 2.001; (1 + 0.1%)^2304 = 10.0026; and (1 + 0.1%)^4608 = 100.053

You can write down the equation (1 + 0.1%)^X = 2, etc.; take logs to get X[ln(1+0.1%)] = ln(2), etc. and solve for:

X = ln(2)/ln(1+0.1%) = 693.404
X = ln(10)/ln(1+0.1%) = 2303.74
X = ln(100)/ln(1+0.1%) = 4607.47

2) Suppose that some quantity grows at a steady pace of 2% per year. About how long does it take to double? About how long does it take to grow tenfold? A hundredfold?

You can go to, for example, http://wolframalpha.com and start entering expressions like (1 + 2%)^10, (1 + 2%)^100, and so on until you get (1 + 2%)^35 = 1.9989; (1 + 2%)^116 = 9.94535; and (1 + 2%)^233 = 100.808

You can write down the equation (1 + 2%)^X = 2, etc.; take logs to get X[ln(1+2%)] = ln(2), etc. and solve for:

X = ln(2)/ln(1+2%) = 35.0028
X = ln(10)/ln(1+2%) = 116.277
X = ln(100)/ln(1+2%) = 232.553

3) Suppose that some quantity grows at a steady pace of 72% per year. About how long does it take to double? About how long does it take to grow tenfold? A hundredfold?

You can go to, for example, http://wolframalpha.com and start entering expressions like (1 + 7%)^10, and so on until you get (1 + 7%)^10 = 1.967; (1 + 7%)^34 = 9.978; and (1 + 7%)^68 = 99.563

You can write down the equation (1 + 7%)^X = 2, etc.; take logs to get X[ln(1+7%)] = ln(2), etc. and solve for:

X = ln(2)/ln(1+7%) = 10.2408
X = ln(10)/ln(1+7%) = 34.0324
X = ln(100)/ln(1+7%) = 68.0648

4) Any patterns or rules of thumb about compound growth and interest jump out at you?

0.1 x 693.404 = 69.3404 which is very close to 2 x 35.0028 which is very close to 7 x 10.2408: take 70 and divide it by the growth rate in percent per year and you get very close to the number of years needed for a compound-growth process to double.

0.1 x 2303.74 = 230.374 which is very close to 2 x 116.277 which is very close to 7 x 34.0324: take 230 and divide it by the growth rate in percent per year and you get very close to the number of years needed for a compound-growth process to multiply itself tenfold.

5) Consider our very long-run global economic real GDP growth and prosperity table from lecture:

In the Shadow of Malthus

Year	Population (Millions)	GDP per Capita ($2015)	Total World GDP ($2015 Billions)
-8000	5	$750	$4
-1000	50	$750	$38
0	170	$750	$128
1500	500	$750	$375
1800	750	$1000	$750
1900	1500	$2000	$3000
2000	6200	$7700	$47740
2015	7400	$10000	$74000

How fast, in percent per year did total world real GDP grow between -8000 and -1000? Between -1000 and the year zero? Between 0 and 1500? Between 1500 and 1800? Between 1800 and 1900? Between 1900 and 2000? Between 2000 and 2015?

Easiest to solve by taking logs:

0.032%/year over -8000 to -1000
0.121%/year over -1000 to 0
0.072%/year over 0 to 1500
0.231%/year over 1500 to 1800
1.386%/year over 1800 to 1900
2.767%/year over 1900 to 2000
2.921%/year over 2000 to 2015

6) How much longer can and is this likely to go on, where "this" is the pace of population and production growth worldwide we have seen since 1900?

Not a question that admits of a definitive answer—we are interested in their thoughts…