
J. Bradford DeLong

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Dear Mark--

The question of the relationship between the social value of an intellectual property-possessing monopolist (like Microsoft) and its total sales depends on (a) the extent to which its sales are pulled away from competitors whose products are close substitutes and (b) whether the monopolist's existence accelerates innovation (by providing shoulders on which others can stand) or by creating a "death zone" in which nobody else dares innovate. Barro doesn't consider the second set of issues at all. And he argues that the ratio of Microsoft's social value to its total sales is equal to one no matter how closely its products are substitutes for the products of others.

I think I have got what's going on in <http://economistsview.typepad.com/economistsview/2007/06/robert-barro-sk.html#comment-73987582>, if I can still do math...

The way I think about these models of "ideas," the setup is like this:

Final goods Y are produced in a competitive market by small firms using the known production function:

\[ Y = AL^{1-\alpha} I^\alpha \]

Where I is a Dixit-Stiglitz aggregation of N varieties of monopoly-produced intermediate goods:

\[ I = \left[ \sum_{i=1}^{N} (x_i)^{\alpha} \right]^{1/(1-\alpha)} \]
Where \( x(j) \) is the amount of final goods purchased by monopolist \( j \) to produce the \( j \)th intermediate good, and \( \sigma \) is between 0 and 1. The \( j \)th monopolist owns the "idea" of and is the exclusive producer of the \( j \)th variety.

Thus the more ideas--the greater the number \( N \) of varieties of intermediate goods that the economy can produce--the better. And the more distinct are the varieties--the less they are substitutes--the smaller is \( \sigma \)--the better. If \( X \) are the total inputs devoted to producing intermediate goods, then the aggregate quantity of intermediate goods supplied to the competitive final-goods producers is:

\[
I = N^{[(1-\alpha)/\alpha]}X
\]

When \( \sigma \) is near zero, the number of varieties matters a lot. When \( \sigma \) is one, the number of varieties doesn't matter for society's productive potential at all.

Now suppose we have an economy that spends a fraction \( \phi \) of gross final output on intermediate goods:

\[
X = \phi Y
\]

Then net final output \( C = Y - X \) is:

\[
C = (1 - \phi)A^{[(1-\alpha)/\alpha]} \phi^{(\sigma(\alpha-\sigma))} LN^{[(\alpha(1-\alpha))]/(1-\alpha)\sigma^2} \]

And the value of inventing a marginal additional variety \( dN \) is:

\[
dC = \left[ \frac{\alpha(1-\sigma)}{(1-\alpha)\sigma} \left( \frac{C}{N} \right) \right] dN
\]

Barro wants to argue that the social value \( dC \) of inventing the marginal \( N \)th variety is equal to the total spending \( dS \) on that marginal \( N \)th variety. With the monopolist charging a price \( P \) for that new variety, total spending on that newly-invented marginal intermediate good is:

\[
dS = \frac{PX}{N} dN = \frac{\phi C}{(1-\phi)N} dN
\]
(Note: dS is *not* the increase in intermediate goods spending. dS is spending on the additional possible intermediate good.) And the ratio of social benefit to total sales of the intermediate good is:

\[
\frac{dC}{dS} = \frac{\alpha(1-\sigma)(1-\varphi)}{(1-\alpha)\sigma P\varphi}
\]

Barro [http://economistsview.typepad.com/economistsview/2007/06/robert-barrosk.html#comment-73987582] uses profit maximization by monopoly intermediate-goods producers with intellectual property rights over ideas to set:

\[
P\varphi = \alpha
\]

\[
\varphi = \alpha\sigma
\]

Which in this standard setup would give us:

\[
\frac{dC}{dS} = \frac{(1-\sigma)(1-\alpha\sigma)}{\sigma(1-\alpha)}
\]

As you can see, as sigma approaches zero the ratio of the social benefit from to the sales of the marginal intermediate goods variety becomes arbitrarily large: for sigma near zero, Microsoft's sales vastly understate its social value. As sigma approaches one, the ratio of the social benefit from to the sales of the marginal intermediate goods variety approaches zero: Microsoft's sales vastly overstate its social value. This is how it should be: an invention that does something completely new (sigma near zero) should have a much bigger impact than an invention that is a close substitute for already existing technologies (sigma near one).

In the limit in which sigma=1, there is--in conventional models--no benefit at all to learning how to produce a new variety. When sigma=1:

\[
I = N^{(1-\sigma)/\alpha}X = X
\]

And net final output does not depend on N:

\[
C = (1-\varphi)^{(\alpha\sigma)/\alpha}A^{(\alpha\sigma)/\alpha}L
\]

So \(dC/dS = 0\).

Barro deviates from standard versions of this "ideas" setup by asserting that $dC/dS$ is independent of sigma and that $dC/dS = 1$. The reason he calculates that $dC/dS$ does not depend on sigma is the particular specification of the total factor productivity term $A$ in his production function. Barro makes total factor productivity in final goods production a function of $N$:

$$A = A_0 N^{((\sigma - \alpha)/\sigma)}$$

Thus as sigma approaches one in Barro's setup, all firms in the economy become more productive and more efficient as a result of the invention of a new intermediate goods variety--no matter how much of or whether they actually use that new intermediate goods variety. Simply the fact that it is possible to produce it provides a big boost to the economy. That's why--in Barro's model--the ratio $dC/dS$ does not depend on whether the new variety is a close substitute for existing varieties or something radically new.