Theory of Economic Growth: Two More Notes

I. Important Equations
I’m going to ask you to memorize five equations that make the Solow growth model truly useful as a cookbook:

\begin{align*}
L_t &= L_0 e^{nt} \\
E_t &= E_0 e^{gt} \\
s &= \frac{s}{n + g + \delta} \\
\kappa^* &= \kappa^* + (\kappa_0 - \kappa^*) e^{-(1-\alpha)(n + g + \delta)t} \\
\frac{Y_t}{L_t} &= (K_t)^{\frac{\alpha}{1-\alpha}} E_t
\end{align*}

Keep these at your fingertips.

II. Increases in g
A very good question today:

An increase in the efficiency-of-labor growth rate \( g \) (a) lowers the steady-state balanced-growth capital-output ratio \( \kappa^* \) and (b) raises the growth path of the efficiency of labor. Which effect dominates when?

An increase in the efficiency-of-labor growth rate \( g \) always raises output per worker: the second effect always dominates.
To see this, write:

\[
\frac{Y_t}{L_t} = (\kappa_t)^{\alpha} E_t,
\]

\[
\frac{d\ln(Y_t/L_t)}{dt} = \frac{\alpha}{1 - \alpha} \frac{d\ln(\kappa_t)}{dt} + \frac{d\ln(E_t)}{dt}
\]

\[
\frac{d\ln(Y_t/L_t)}{dt} = \frac{\alpha}{1 - \alpha} \left[ \frac{s}{\kappa} - (n + g + \delta) \right] + g
\]

\[
\frac{d\ln(Y_t/L_t)}{dt} = \alpha \left[ \frac{s}{\kappa} - (n + \delta) \right] + (1 - \alpha)g
\]

and:

\[
\frac{d\ln(\kappa_t)}{dt} = (1 - \alpha) \left[ \frac{s}{\kappa} - (n + g + \delta) \right]
\]

Now consider a sudden, discontinuous rise in g. It will raise the instantaneous growth rate of output per worker, and lower the instantaneous growth rate of the capital-output ratio. After a short period of time, output per worker will be higher and the capital-output ratio lower than if g had remained constant. The lower capital-output ratio will further raise the growth rate of output per worker, and widen the gap even more than the difference in g.

We have demonstrated that:

- At time zero, the growth rate of output per worker \(Y/L\) is greater with the upward shift in g than it would have been otherwise.
- At all times, the growth rate of the capital-output ratio \(\kappa\) is less with the upward shift in g than it would have been otherwise.
• At all times after time zero, the capital-output ratio is lower with the upward shift in \( g \) than it would have been otherwise.
• At all times after time zero, the growth rate of output per worker is greater with a lower capital-output ratio and a higher value of \( g \) than it is with a higher capital-output ratio and a lower value of \( g \).

Thus even though a rise in \( g \) lowers the steady-state balanced-growth capital-output ratio and lowers the initial value of steady-state output per worker, it is always good for the economy.