Lecture Notes: Econ 101B: August 31 2006

Theory of Economic Growth: Two More Notes

I. Important Equations

I'm going to ask you to memorize five equations that make the Solow growth model truly useful as a cookbook:

$$L_t = L_0 e^{nt}$$

$$E_t = E_0 e^{gt}$$

$$\kappa^* = \frac{s}{n + g + \delta}$$

$$\kappa_t = \kappa^* + (\kappa_0 - \kappa^*) e^{-(1-\alpha)(n+g+\delta)t}$$

$$\frac{Y_t}{L_t} = (\kappa_t)^{\frac{\alpha}{1-\alpha}} E_t$$

Keep these at your fingertips.

II. Increases in g

A very good question today:

> An increase in the efficiency-of-labor growth rate g (a) lowers the steady-state balanced-growth capital-output ratio k* and (b) raises the growth path of the efficiency of labor. Which effect dominates when?

An increase in the efficiency-of-labor growth rate g always raises output per worker: the second effect always dominates.

To see this, write:

$$\frac{Y_t}{L_t} = (\kappa_t)^{\frac{\alpha}{1-\alpha}} E_t$$

$$\frac{d\ln(Y_t/L_t)}{dt} = \frac{\alpha}{1-\alpha}\frac{d\ln(\kappa_t)}{dt} + \frac{d\ln(E_t)}{dt}$$

$$\frac{d\ln(Y_t/L_t)}{dt} = \frac{\alpha}{1-\alpha}(1-\alpha)\left[\frac{s}{\kappa} - (n+g+\delta)\right] + g$$

$$\frac{d\ln(Y_t/L_t)}{dt} = \alpha\left[\frac{s}{\kappa} - (n+\delta)\right] + (1-\alpha)g$$

and:

$$\frac{d\ln(\kappa_t)}{dt} = (1-\alpha)\left[\frac{s}{\kappa} - (n+g+\delta)\right]$$

Now consider a sudden, discontinuous rise in g. It will raise the instantaneous growth rate of output per worker, and lower the instantaneous growth rate of the capital-output ratio. After a short period of time, output per worker will be higher and the capital-output ratio lower than if g had remained constant. The lower capital-output ratio will further raise the growth rate of output per worker, and widen the gap even more than the difference in g.

We have demonstrated that:

- At time zero, the growth rate of output per worker Y/L is greater with the upward shift in g than it would have been otherwise.
- At all times, the growth rate of the capital-output ratio κ is less with the upward shift in g than it would have been otherwise.

- At all times after time zero, the capital-output ratio is lower with the upward shift in g than it would have been otherwise.
- At all times after time zero, the growth rate of output per worker is greater with a lower capital-output ratio and a higher value of g than it is with a higher capital-output ratio and a lower value of g.

Thus even though a rise in g *lowers* the steady-state balanced-growth capital-output ratio and *lowers* the initial value of steady-state output per worker, it is always good for the economy.