Lecture Notes: Econ 101B: September 5, 2006

Explorations in the Theory of Economic Growth

From Last Time: We Now Have a Base Camp

We can now explore in all directions: modifying our model to add one effect or another, calculating the consequences of complicating the model in that way, and then returning to our base case to conduct another exploration.

Explorations:

- Model with natural resources
- Model with resource-augmenting (rather than labor-augmenting) productivity growth
- Model with Malthusian population dynamics

That's all I think we will have time for today.

Natural Resources: Set Up

Let's add natural resources, R, to our model. Make it a part of our production function. Also, divide our technology-organization productivity measure E into two parts: E_L, the efficiency with which the economy can use labor, and E_R, Our production function is thus:

$$Y = K^{\alpha}(E_L L)^{1-\alpha-\beta}(E_R R)^{\beta}$$

We have our standard behavioral relationships:

Households do their things: The labor force L grows at a constant proportional rate n:

$$\frac{d\ln(L)}{dt} = n \quad \Rightarrow \quad L_t = L_0 e^{nt}$$

Scientists and entrepreneurs do their thing The efficiency of labor E_L grows at a constant proportional rate g_L:

$$\frac{d\ln\left(E_L\right)}{dt} = g_L \quad \Rightarrow \quad E_{Lt} = E_{L0} e^{g_L t}$$

Executives and investors do their thing. A fraction δ of the capital stock wears out each year, but society saves and invests a fraction s of output Y:

$$\frac{d\ln(K)}{dt} = s\left(\frac{Y}{K}\right) - \delta = \frac{s}{\kappa} - \delta \quad \Rightarrow \quad K_t = ??? \quad \kappa_t = ???$$

And we have two extra rules:

The stock of natural resources R is fixed at R_0, for all time. And to simplify equations a bit, we will set $R_0=1$:

$$\frac{d\ln\left(R\right)}{dt} = 0 \quad \Rightarrow \quad R_t = R_0 = 1$$

The efficiency with which the economy can use natural resources grows at a constant proportional rate g_R:

$$\frac{d\ln\left(E_R\right)}{dt} = g_R \quad \Rightarrow \quad E_{Rt} = E_{R0} e^{g_R t}$$

Everybody keeps on doing their thing from time zero to the end of time.

Natural Resources: Consequences

With the augmented production function, the proportional growth rate of output is:

$$\frac{d\ln(Y)}{dt} = \alpha\frac{d\ln(K)}{dt} + (1-\alpha-\beta)\frac{d\ln(L)}{dt} + (1-\alpha-\beta)\frac{d\ln(E_L)}{dt} + \beta\frac{d\ln(E_R)}{dt}$$
$$\frac{d\ln(Y)}{dt} = \alpha\frac{d\ln(K)}{dt} + (1-\alpha-\beta)n + (1-\alpha-\beta)g_L + \beta g_R$$

the proportional growth rate of the capital-output ratio κ is:

$$\frac{d\ln(\kappa)}{dt} = (1-\alpha)\frac{d\ln(K)}{dt} - (1-\alpha-\beta)n - (1-\alpha-\beta)g_L - \beta g_R$$

which gives us a steady-state balanced-growth capital-output ratio of:

$$\kappa^* = \frac{s}{\delta + \left(\frac{1-\alpha-\beta}{1-\alpha}\right)n + \left(\frac{1-\alpha-\beta}{1-\alpha}\right)g_L + \left(\frac{\beta}{1-\alpha}\right)g_R}$$

If we take the weighted average of the two efficiency growth rates:

$$g = \left(\frac{1-\alpha-\beta}{1-\alpha}\right)g_L + \left(\frac{\beta}{1-\alpha}\right)g_R$$

We can write the steady-state balanced-growth capital-output ratio as:

$$\kappa^* = \frac{s}{\delta + \left(\frac{1-\alpha-\beta}{1-\alpha}\right)n + g}$$

In this version of the model, it doesn't really matter where exactly increased efficiency comes from.

Considering the rate of growth of output:

$$\frac{d\ln(Y)}{dt} = \alpha\frac{d\ln(K)}{dt} + (1-\alpha-\beta)n + (1-\alpha-\beta)g_L + \beta g_R$$

Substituting κY for K, and solving for the growth rate of otput per worker, Y/L:

$$\frac{d\ln(Y/L)}{dt} = \left(\frac{\alpha}{1-\alpha}\right)\frac{d\ln(\kappa)}{dt} + g - \left(\frac{\beta}{1-\alpha}\right)n$$

Tells us that along the economy's steady-state balanced-growth path, the growth rate of output per worker is:

$$\frac{d\ln(Y/L)}{dt} = g - \left(\frac{\beta}{1-\alpha}\right)n$$

Natural resources do make a difference. They make faster population growth a much greater drag on increasing prosperity—depending how large β is.

Malthusian Nightmares: Set Up

Let's take our model of natural resources. For convenience' sake only set g_L=0 and E_{Lt}= E_{Lt}=1. And let's assume that the richer is the economy, the faster is population and labor-force growth:

$$n_t = \gamma\left[\left(\frac{Y_t}{L_t}\right) - y^*\right]$$

And let's look for an equilibrium in which the capital-labor ratio is constant, output per worker is constant, and the labor force is growing at the same rate as the growth rate g_R of the efficiency with which the economy uses natural resources.

Malthusian Nightmares: Consequences

Then:

$$n = g_R$$

$$\frac{Y}{L} = y^* + \frac{g_R}{\gamma}$$

Substituting into the expression for the steady-state capital-output ratio:

$$\kappa^* = \frac{s}{\left(\frac{1-\alpha-\beta}{1-\alpha}\right)n + \delta + \left(\frac{\beta}{1-\alpha}\right)g_R} = \frac{s}{\delta + g_R}$$

Since output per worker along the steady-state growth path equals:

$$\frac{Y}{L} = (\kappa^*)^{\frac{\alpha}{1-\alpha}}(E_R)^{\frac{\beta}{1-\alpha}}(L)^{\frac{-\beta}{1-\alpha}}$$

We can solve for E_R/L:

$$\left(\frac{E_R}{L}\right) = \left(y^* + \frac{g_R}{\gamma}\right)^{\frac{1-\alpha}{\beta}}\left(\frac{g_R + \delta}{s}\right)^{\frac{\alpha}{\beta}}$$

and then for L:

$$L = E_R\left(\frac{\gamma}{\gamma y^* + g_R}\right)^{\frac{1-\alpha}{\beta}}\left(\frac{s}{g_R + \delta}\right)^{\frac{\alpha}{\beta}}$$

In this equilibrium, the ratio of the efficiency with which resources are used to the labor force is just enough to give a standard of living at which the population can grow at g_R. If g_R is low, then society will be very poor. Increases in g_R produce faster growth in and eventually larger populations, not higher living standards and productivity levels.

How do you make a better society from one caught in this Malthusian trap? The most obvious way would be to figure out how to raise y*--but there are limits to the effectiveness of that strategy.

How do you make a happy society? How did we escape from this Malthusian trap—in which most of the human race was held between 6000 BC and 1650 or so? That's a complex and still poorly understood story.