## Lecture Notes: Econ 101B: September 14, 2006

# Modeling the High-Tech Computer Revolution of the 1990s

Suppose we have two types of capital: normal capital K, and high-tech capital T, with the production function:

$$Y = K^{\alpha} T^{\phi} (EL)^{1-\alpha-\phi}$$

And capital accumulation equations:

$$\frac{dK}{dt} = s_K Y - \delta_K K$$
$$\frac{dT}{dt} = \frac{s_T Y}{p_T} - \delta_T T$$

The high-tech computer revolution here takes the form of a rapidly-falling price $p_T$ of high-tech capital, where:

$$\frac{d\ln(p_T)}{dt} = -\eta$$

It turns out to be more convenient to think not of T, but of $p_T T$—not of the size of the high-tech capital stock, but of its value. The rate of growth of $p_T T$ has two terms: T increases, and $p_T$ falls. So:

$$\frac{d\ln(p_T T)}{dt} = s_T \left( \frac{Y}{p_K T} \right) - \delta_T - \eta$$

And writing the production function in terms of the value of high-tech capital gives us:

$$Y = K^{\alpha}(p_T T)^{\phi} L^{1-\alpha-\phi} E^{1-\alpha-\phi} p_K^{-\phi}$$

$$\frac{d\ln(Y)}{dt} = \alpha\left(s_K\left(\frac{Y}{K}\right) - \delta_K\right) + \phi\left(s_T\left(\frac{Y}{p_T T}\right) - \delta_T - \eta\right) + (1-\alpha-\phi)n + (1-\alpha-\phi)g + \phi\eta$$

This will give us a steady-state rate of growth along the balanced-growth path:

$$\frac{d\ln(Y/L)}{dt} = g + \frac{\phi\eta}{(1-\alpha-\phi)}$$

and steady-state capital-value to output ratios:

$$\frac{p_T T}{Y} = \frac{s_T}{n + \left(g + \dfrac{\phi\eta}{(1-\alpha-\phi)}\right) + \left(\delta_T + \eta\right)}$$

$$\frac{K}{Y} = \frac{s_K}{n + \left(g + \dfrac{\phi\eta}{(1-\alpha-\phi)}\right) + \delta_K}$$

Note: the speed of convergence of the value high-tech capital-output ratio to its steady-state value will be rapid… 1/e times on the order of a decade…

Note: understand what  happened in the late 1990s as sudden jumps in both η and ϕ…

Note: where Marx went wrong: confusing the value with the physical capital-output ratio…