A Graduate Teaching Note:

On Robert Barro's (2005) "Rare Events and the Equity Premium" and T.A. Rietz's (1988) "The Equity Risk Premium: A Solution"

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Summary:

Robert Barro's 2005 "Rare Events and the Equity Premium" paper <http://papers.nber.org/papers/w11310.pdf> argues that Rietz (1988), "The Equity Risk Premium: A Solution" <http://www.biz.uiowa.edu/faculty/trietz/> was correct: it is the possibility of future disastrous falls in economy-wide consumption and payouts on equities that explains the high equity premium.

The primary channel in the models of Rietz and Barro, however, is not that fear of future catastrophe depresses the price and hence raise the in-sample average return to equities (for we don't observe such disastrous falls in our sample). The primary channel is that fear of future catastrophe raises desired savings to carry purchasing power forward in time in case of need. Since assets are in fixed supply in the Lucas-tree model Barro uses, this outward shift in savings demand drives the prices of real bonds up and the returns on real bonds down. It also--for coefficients of relative risk aversion greater than one--drives the prices up and the returns

down on equities as well. A greater fear of future catastrophe is a source of high, not low, price-dividend and price-earnings ratios.

This is worth noting for three reasons:

This is worth noting, first, because Rietz gets the logic of his model wrong. He writes:

> The motivation for adding the third [macroeconomic catastrophe] state [to the transition matrix] is simple. Risk-averse equity holders demand a high return to compensate for the extreme losses they may incur during an unlikely, but severe, market crash. To the extent that equity returns have been high with no crashes, equity owners have been compensated for the crashes that happen not to occur. High risk premia should not be puzzling in such a world...

By contrast, Barro has it right:

> Less intuitively, a rise in [the probability of catastrophe] also lowers the rate of return on the risky asset.... [If γ] > 1, this change reflects partly an *increase* in the price of equity.... [A] rise in [the probability of catastrophe] motivates a shift toward the risk-free asset and away from the risky one--this force would lower the equity price. However, households are also motivated to hold more assets overall because of greater uncertainty about the future. If [γ] > 1, this second force dominates, leading to a net increase in the equity price.... In any event, the risk-free rate falls by more than the risky rate, so that the [equity premium] increases...

This is worth noting, second, because I find this story implicit in Rietz (1988) and Barro (2005) to be implausible, albeit ingenious. The implicit story is that high stock prices (in terms of consumption goods) are a sign of a relatively high chance of macroeconomic catastrophe. Stock market multiples were much higher in 1999 than they were in 1982 and much higher in 1929 than in 1922 because the likelihood of a macroeconomic catastrophe was much higher in 1999 and 1929 than it was in 1982 and 1922. People were desperate to save for the future to insure against the greater likelihood of macroeconomic catastrophe, and that drove up stock prices. This doesn't seem to me to be an admissible reading of the psychological evidence: investors appear to have been expecting high, not low, returns in the late 1920s and 1990s.

This is worth noting, third, because this seems to me to be a trap set for us by our habit of using the Lucas-tree model of Lucas (1978), "Asset Prices in an Exchange Economy" <http://links.jstor.org/sici?sici=0012-9682%28197811%2946%3A6%3C1429%3AAAPIAEE%3E2.0.CO%3B2-I> as a workhorse. The Lucas-tree model has neither production nor accumulation. This makes it easy to solve. This makes its responses to many shocks perverse--since agents cannot respond to shocks by raising (or lowering) production or by accumulating (or decumulating), prices must move to make it so that they do not wish to do so.

But let's take a look at the guts of a simplified version of the Lucas-tree model to understand how Rietz and Barro's solution to the equity premium puzzle works:

The Stripped-Down Lucas-Tree Model: Basics:

The Setup:

Consider an economy with a single representative consumer who seeks to maximize:

$$E\left\{\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^{t} U(C_t)\right\}$$

where C_t is the stochastic process for consumption in period t, ρ is the discount factor, U(•) is the single-period utility function, and E{•} is an expectation. Production is entirely exogenous: there is no possibility of any action affecting the level of output at any time. Output is not storable: consumption is thus also exogenous. Thus there is neither a production nor an accumulation decision to be made--or, rather, asset prices in this model move so that the representative consumer does not wish to accumulate.

We are going to consider a particular very simple constant relative risk aversion utility function:

$$U(C_t) = 1 - \frac{1}{C_t}$$

which is simply the constant relative risk aversion utility function:

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma}$$

with γ=2. Start out with a very simple structure for the evolution of C_t:

$$C_{t+1} = \begin{cases} \dfrac{C_t(1+g)}{1-\sigma} & \text{with probability } 1/2 \\ \dfrac{C_t(1+g)}{1+\sigma} & \text{with probability } 1/2 \end{cases}$$

Now let's consider two asset prices in this model, the price P_t^f of a one-period security that pays off one unit in period t+1 in all states of the world--a riskfree security--and the price P_t^o of a one-period risky equity security that pays off C_{t+1} units in period t+1.

Prices and Returns:

The first-order condition for the riskfree security for an agent with constant relative risk aversion is:

$$0 = \frac{-P_t^f}{C_t^\gamma} + \left(\frac{1}{1+\rho}\right)E\left\{\frac{1}{C_{t+1}^\gamma}\right\}$$

The representative agent must be indifferent between consuming the amount P_t^f now and saving it in the form of the riskfree asset until next period and then consuming the proceeds. Substituting in the possible outcomes and specializing to the case γ=2, we have:

$$P_t^f = \left(\frac{1}{1+\rho}\right) E\left\{\frac{1}{(C_{t+1}/C_t)^2}\right\} = \left(\frac{1}{1+\rho}\right)\left[\frac{1/2}{\left(\frac{1+g}{1-\sigma}\right)^2} + \frac{1/2}{\left(\frac{1+g}{1+\sigma}\right)^2}\right]$$

Thus we can calculate the price of the riskfree asset:

$$P_t^f = \left(\frac{1}{1+\rho}\right)\left\{\frac{(1+\sigma^2)}{(1+g)^2}\right\} \quad for \quad \gamma = 2$$

And determine the one-period riskfree gross interest rate:

$$R_t^f = \frac{1}{P_t^f} = \frac{(1+\rho)(1+g)^2}{(1+\sigma^2)} \quad for \quad \gamma = 2$$

For the one-period risky equity security, the first-order condition is:

$$0 = \frac{-P_t^o}{C_t^\gamma} + \left(\frac{1}{1+\rho}\right) E\left\{\frac{C_{t+1}}{C_{t+1}^\gamma}\right\}$$

The representative agent must be indifferent between consuming the amount P_t^o now and saving it in the form of one-period risky equities and consuming the proceeds next period. From this we can calculate that the pricing solution is:

$$\frac{P_t^o}{C_t} = \left(\frac{1}{1+\rho}\right)\left\{\frac{1}{(1+g)}\right\} \quad for \quad \gamma = 2$$

with the associated expected gross rate of return:

$$ER_t^e = \left(\frac{(1+g)^2(1+\rho)}{1-\sigma^2}\right) \quad for \quad \gamma = 2$$

The gross equity return premium--the ratio of expected returns on equity to bonds--is:

$$\frac{ER_t^e}{R_t^f} = \frac{1+\sigma^2}{1-\sigma^2} \quad for \quad \gamma = 2$$

Long-Duration Equities:

Now let's consider the price of in period t of an infinitely-lived equity claim--one that pays off C_{t+1} in period t+1, C_{t+2} in period t+2, and so on. CRRA plus C_t being the only state variable give us that such an infinitely-lived equity will trade as a function of the one-period equity:

$$P_t^e = \pi P_t^o$$

Thus prices of short- and long-term equities and the level of consumption are perfectly correlated. This means that rates of return will be equalized across one-period and long-lived equities, and so:

$$\frac{C_{t+1}}{P_t^o} = \frac{C_{t+1} + \pi P_{t+1}^o}{\pi P_t^o}$$

which can be solved to yield:

$$\pi = \frac{1}{1-(P_t^o / C_t)}$$

and:

$$P_t^e = \frac{P_t^o}{1-(P_t^o / C_t)}$$

Discussion:

Here we have the basics of the model. We can solve it for sample parameter values--a scratchpad for doing this for ρ, g, and σ (with γ=2) is at: "Equity Premium Project: Simple Lucas-Tree Spreadsheet for Barro-Rietz Critique" <http://spreadsheets.google.com/pub?key=p_zylRhg4towFfq53r00UBA>.

And here we can see the equity premium puzzle. For those who view the task as being to explain the comovements of asset prices and economic fundamentals as assessed by average levels of consumption spending, the task is to combine a σ of approximately .03 with a gross riskfree rate on the order of 1.01 and a gross equity return on the order of 1.07. This task is impossible For those who view the task as being to explain the comovements of asset prices and are willing to assume that relevant investors' consumption risk cannot be spread out to the economy as a whole, the task is to combine a σ of approximately .15 with a gross riskfree rate on the order of 1.01 and a gross equity return on the order of 1.07. This is very, very difficult.

It is the problem that the introduction of a small chance of a catastrophe--a catastrophe not seen in sample--is supposed to solve, or at least reduce.

Adding a Chance of Catastrophe:

Now let's add what Rietz (1988) introduced into the problem and what Barro (2005) calls V-events: a k probability chance that consumption and equity returns both drop to a fraction theta of their levels, and then resumes their standard stochastic growth process, but riskfree assets retain their returns. The canonical V-event is a Great Depression, as opposed to a revolution or war or financial crisis or other catastrophe that triggers substantial inflation or confiscation, and thus impairs real debt as well as equity.

We model V-events as did Rietz (1988): we simply add a third element to the state-transition vector:

$$C_{t+1} = \begin{cases} \dfrac{C_t(1+g)}{1-\sigma} & with \;\; probability \;\; (1-k)/2 \\ \dfrac{C_t(1+g)}{1+\sigma} & with \;\; probability \;\; (1-k)/2 \\ \theta C_t & with \;\; probability \;\; k \end{cases}$$

Now let's calculate the price of the riskfree asset with this new set of transition probabilities. We find that the introduction of this steep fall price of the riskfree asset rises by a lot--an amount proportional to k, and inversely proportional to θ squared:

$$P_t^{fV}(k,\theta) = (1-k)P_t^f + \left(\frac{1}{1+\rho}\right)\frac{k}{\theta^2}$$

Even if we remain in states of the world in which we never observe such a V-type catastrophe, the riskfree rate drops significantly. The effect on riskfree bond prices is proportional to the chance of catastrophe, and inversely proportional to the parameter theta that quantifies the size of the catastrophe raised to the power of the coefficient of relative risk aversion. Our coefficient of relative risk aversion of 2 is not that favorable to the Rietz-Barro theory. A coefficient of 4 or 5 would make it much easier to account for the low riskfree rate.

Once we add the chance of a V-event, the price of the one-period-ahead equity claim jumps as well (for it to fall you would have to have agents with relative risk aversion less than one):

$$\frac{P_t^{oV}}{C_t} = (1-k)\frac{P_t^o}{C_t} + k\left[\left(\frac{1}{1+\rho}\right)\frac{1}{\theta}\right]$$

But only by an amount inversely proportional to θ. So the equity premium return widens. Since the price of long-lived equities goes with:

$$P_t^e = \frac{P_t^o}{1 - P_t^o / C_t}$$

it too rises with a chance of future disaster. As long as the coefficient of relative risk aversion is greater than one, a possibility of future disaster is "good news" for the stock market: It increases the incentive to save. But in the Lucas-tree model assets are in fixed supply. So an increase in the savings-driven demand for assets increases their price.

In addition to V-events, we have W-events: events that reduce consumption and dividends but also reduce bond returns to the same degree. Such W-events affect the prices of both equity-like claims and bonds o the same degree, and to first order the equity premium is unaffected:

$$P_t^{fW}(k,\theta) = (1-k)P_t^f + \left(\frac{1}{1+\rho}\right)\frac{k}{\theta}$$

$$\frac{P_t^{oW}}{C_t} = (1-k)\frac{P_t^o}{C_t} + k\left[\left(\frac{1}{1+\rho}\right)\frac{1}{\theta}\right]$$

$$P_t^e = \frac{P_t^o}{1-(P_t^o/C_t)}$$

Once again, a basic spreadsheet to play with is at: "Equity Premium Project: Simple Lucas-Tree Spreadsheet for Barro-Rietz Critique" <http://spreadsheets.google.com/pub?key=p_zylRhg4towFfq53r00UBA>.

It is worth noting that most catastrophes--international financial crises that generate high inflation, wars, revolutions, negative technological shocks that trigger both reductions in output and governmental fiscal crisis, et cetera--look a lot more like W-events than like V-events. Indeed, there are many catastrophes that look like Super-W-events: nominal debt values are strongly impaired but real equity values remain real. The canonical V-event is the deflationary catastrophe of the Great Depression itself, and that is definitely in our sample.

Conclusion:

My conclusion? Rietz (1988) and Barro's (2005) invocation of a small chance of a future macroeconomic catastrophe can indeed solve the puzzle of why riskfree rates are so low given the substantial return to risky equities and to productive capital. But it does so at the price of obliging us to carry baggage that we probably do not want to carry.

The baggage is that--as Barro (2005) recognizes but Rietz (1988) did not--the same channel that accounts for the equity premium also carries with it the prediction high stock prices (in terms of consumption goods) are a sign of a relatively high chance of macroeconomic catastrophe. The riskfree rate is low because people are desperate to save for the future to insure against macroeconomic catastrophe. This savings motive drives up stock prices as well as bond prices. And this doesn't seem to me to be a plausible reading of the psychological evidence: investors appear to have been expecting good times--not bad times--in the late 1920s and 1990s.

I think one thing that is going on is that our habit of using the Lucas-tree model of Lucas (1978), "Asset Prices in an Exchange Economy" <http://links.jstor.org/sici?sici=0012-9682%28197811%2946%3A6%3C1429%3AAAPIAEE%3E2.0.CO%3B2-I> as a workhorse has turned out to be a trap. The Lucas-tree model has neither production nor accumulation. This makes it easy to solve. But this makes its responses perverse. There are no scarce resources to be allocated among alternative uses. There are only asset prices which must move so as to make agents unwilling to try to reallocate resources. It is, I think, not surprising that an economic model in which resource allocation plays no role is a dangerous tool to use in trying to understand the world.