Problem Set #4 Solutions

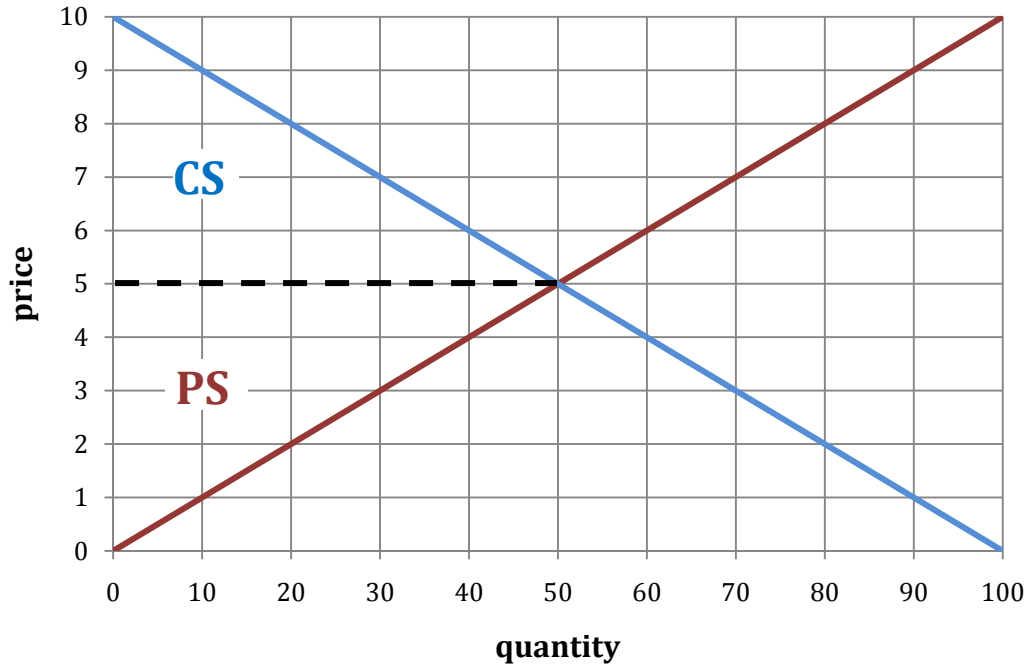
Consider an economy with three types of workers—Dharmas, Egberts, and Gregs—who produce yoga lessons, ceramic plates, and cups of coffee respectively. In this economy the prices of yoga lessons and ceramic plates are expressed in cups of coffee.

Questions 1 through 5 focus on the supply and demand for plates. Suppose that Egberts are willing to produce plates according to the following rule: at a price of zero, they will make zero plates, for every 1-cup-of-coffee increase in the price of plates, they are willing to make ten additional plates.

Note that questions 1 through 5 did not request graphs, and indeed graphs are not needed to solve them. The graphs that appear in these solutions are not necessary, but help to illustrate the respective markets, their equilibria, and attendant consumer and producer surpluses.

Note that in the markets described in questions 1 through 5 price (the vertical dimension in the graph) is expressed in cups of coffee per plate, and quantity (the horizontal dimension) is given in plates. Thus any product of price and quantity (represented by an area on the graph) is measured in units of cups of coffee.

1. Suppose that demand for plates follows this rule: if plates cost 10-cups-of-coffee, nobody wants to buy any. Each 1-cup-of-coffee reduction in the price of plates leads consumers to want to buy an additional ten plates.



a. What is the market equilibrium price of plates in this market?

The quantity supplied is given by the equation: $Q_S = 10P$

The quantity demanded is given by the equation: $Q_D = 100 - 10P$

The equilibrium price, P^*, is the price at which the quantity supplied equals the quantity demanded—where $Q_S = Q_D$. Thus:

$10P^* = 100 - 10P^* \Rightarrow 20P^* = 100 \Rightarrow P^* = 5$

The equilibrium price of plates in this market is 5 cups of coffee.

b. What is the market equilibrium quantity of plates exchanged in this market?

At the equilibrium price, the equilibrium quantity of plates exchanged is given by either the quantity supplied or the quantity demanded: by definition the market equilibrium occurs where these two are the same. Given the equilibrium price found in part a, we have:

$Q_S = 10(5) = 50$

$Q_D = 100 - 10(5) = 100 - 50 = 50$

The equilibrium quantity of plates exchanged in this market is 50.

c. What is the consumer surplus in this market, in units of cups of coffee?

The consumer surplus (CS) is equal to the area of below the demand curve and above the equilibrium price. That is, in this case it is equal to the area of a triangle with height 5 and base 50. The consumer surplus is thus:

$$\frac{5 \cdot 50}{2} = 125$$

The consumer surplus is equal to 125 cups of coffee.

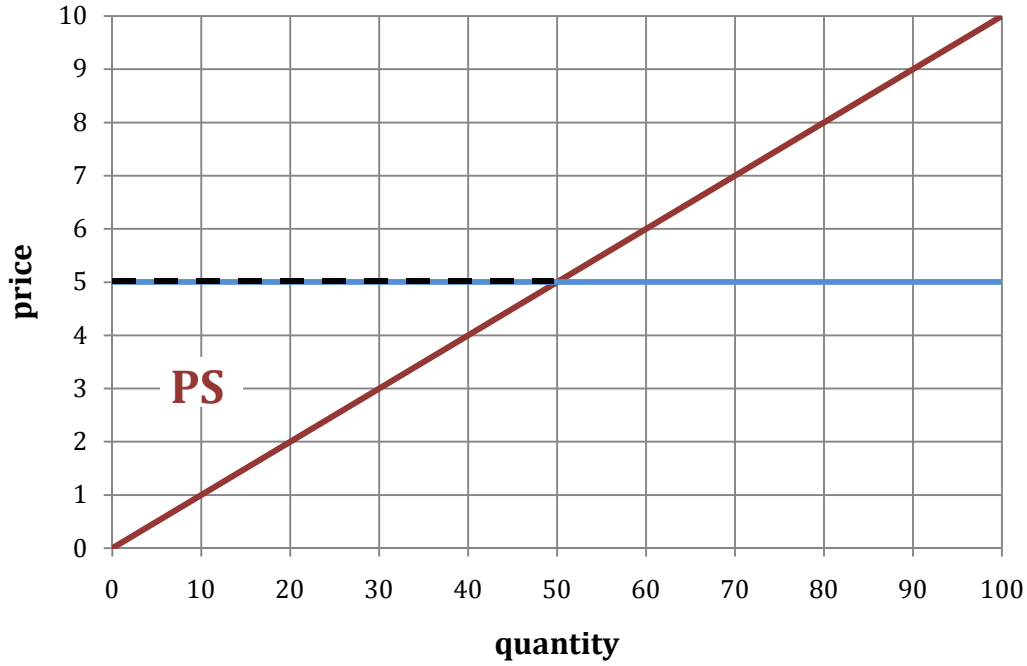
d. What is the producer surplus in this market, in units of cups of coffee?

The producer surplus (PS) is equal to the area of above the supply curve and below the equilibrium price. That is, in this case it is equal to the area of a triangle with height 5 and base 50. The producer surplus is thus:

$$\frac{5 \cdot 50}{2} = 125$$

The producer surplus is equal to 125 cups of coffee.

2. Like (1), but with a different consumer demand: if plates cost more than 5-cups-of-coffee, nobody wants to buy any. However, consumers are willing to buy an unlimited number of plates at a price of 5-cups-of-coffee per plate.



a. What is the market equilibrium price of plates in this market?

The quantity supplied is given by the equation: $Q_S = 10P$

The quantity demanded is given by the equation: $P = 5$

The equilibrium price, P^*, is the price at which the quantity supplied equals the quantity demanded—where $Q_S = Q_D$. But since the quantity demanded function in this case is $P = 5$ it must be that $P^* = 5$.

The equilibrium price of plates in this market is 5 cups of coffee.

b. What is the market equilibrium quantity of plates exchanged in this market?

At the equilibrium price, the equilibrium quantity of plates exchanged is given by either the quantity supplied or the quantity demanded. In this case we have an explicit function for Q_S but not for Q_D, so look at Q_S. Given the equilibrium price found in part a, we have:

$Q_S = 10(5) = 50$

The equilibrium quantity of plates exchanged in this market is 50.

c. What is the consumer surplus in this market, in units of cups of coffee?

The consumer surplus (CS) is equal to the area of below the demand curve and above the equilibrium price. In this market there is no area between the demand curve and the equilibrium price.

The consumer surplus is equal to zero cups of coffee.

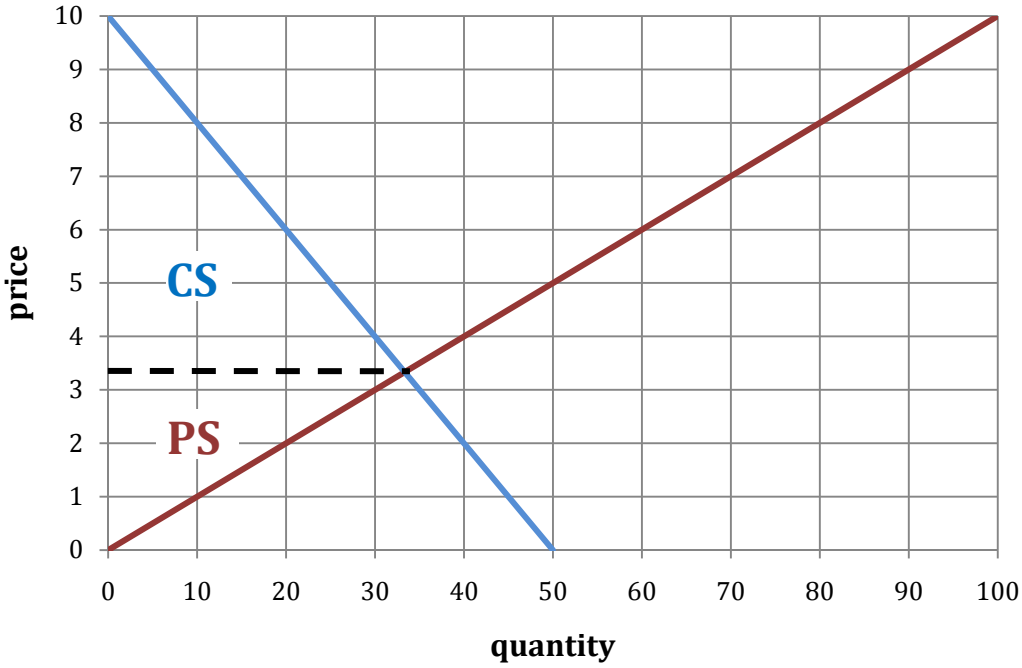
d. What is the producer surplus in this market, in units of cups of coffee?

The producer surplus (PS) is equal to the area of above the supply curve and below the equilibrium price. That is, in this case it is equal to the area of a triangle with height 5 and base 50. The producer surplus is thus:

$$\frac{5 \cdot 50}{2} = 125$$

The producer surplus is equal to 125 cups of coffee.

3. Like (1), but with a different consumer demand: if plates cost 10-cups-of-coffee, nobody wants to buy any. Each 1-cup-of-coffee reduction in the price of plates leads consumers to want to buy an additional five plates.



a. What is the market equilibrium price of plates in this market?

The quantity supplied is given by the equation: $Q_S = 10P$

The quantity demanded is given by the equation: $Q_D = 50 - 5P$

The equilibrium price, P^*, is the price at which the quantity supplied equals the quantity demanded—where $Q_S = Q_D$. Thus:

$$10P^* = 50 - 5P^* \Rightarrow 15P^* = 50 \Rightarrow P^* = \frac{10}{3} = 3\frac{1}{3}$$

The equilibrium price of plates in this market is $3\frac{1}{3}$ cups of coffee.

b. What is the market equilibrium quantity of plates exchanged in this market?

At the equilibrium price, the equilibrium quantity of plates exchanged is given by either the quantity supplied or the quantity demanded: by definition the market equilibrium occurs where these two are the same. Given the equilibrium price found in part a, we have:

$$Q_S = 10(\tfrac{10}{3}) = \frac{100}{3} = 33\frac{1}{3}$$

$$Q_D = 50 - 5(\tfrac{10}{3}) = \frac{150}{3} - \frac{50}{3} = \frac{100}{3} = 33\frac{1}{3}$$

The equilibrium quantity of plates exchanged in this market is $33\frac{1}{3}$.

c. What is the consumer surplus in this market, in units of cups of coffee?

The consumer surplus (CS) is equal to the area of below the demand curve and above the equilibrium price. That is, in this case it is equal to the area of a triangle with height $\frac{20}{3}$ and base $\frac{100}{3}$. The consumer surplus is thus:

$$\frac{1}{2} \cdot \frac{20}{3} \cdot \frac{100}{3} = \frac{1000}{9} = 111\frac{1}{9}$$

The consumer surplus is equal to $111\frac{1}{9}$ cups of coffee.

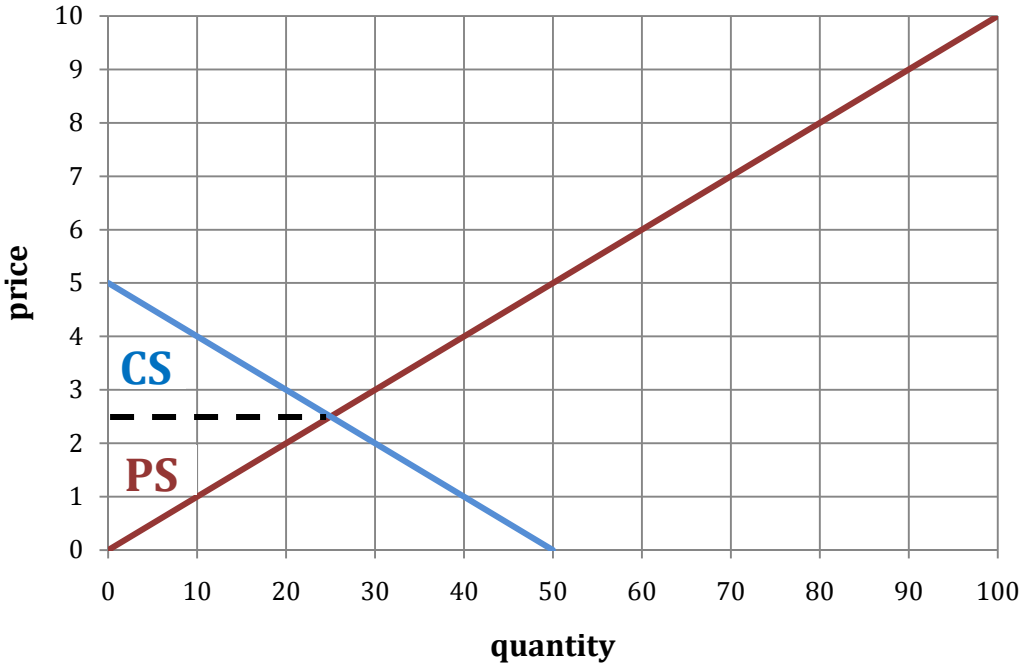
d. What is the producer surplus in this market, in units of cups of coffee?

The producer surplus (PS) is equal to the area of above the supply curve and below the equilibrium price. That is, in this case it is equal to the area of a triangle with height $\frac{10}{3}$ and base $\frac{100}{3}$. The producer surplus is thus:

$$\frac{1}{2} \cdot \frac{10}{3} \cdot \frac{100}{3} = \frac{500}{9} = 55\frac{5}{9}$$

The producer surplus is equal to $55\frac{5}{9}$ cups of coffee.

4. Like (1), but with a different consumer demand: if plates cost 5-cups-of-coffee, nobody wants to buy any. Each 1-cup-of-coffee reduction in the price of plates leads consumers to want to buy an additional ten plates.



a. What is the market equilibrium price of plates in this market?

The quantity supplied is given by the equation: $Q_S = 10P$

The quantity demanded is given by the equation: $Q_D = 50 - 10P$

The equilibrium price, P^*, is the price at which the quantity supplied equals the quantity demanded—where $Q_S = Q_D$. Thus:

$10P^* = 50 - 10P^* \Rightarrow 20P^* = 50 \Rightarrow P^* = 2.5$

The equilibrium price of plates in this market is 2.5 cups of coffee.

b. What is the market equilibrium quantity of plates exchanged in this market?

At the equilibrium price, the equilibrium quantity of plates exchanged is given by either the quantity supplied or the quantity demanded: by definition the market equilibrium occurs where these two are the same. Given the equilibrium price found in part a, we have:

$Q_S = 10(2.5) = 25$

$Q_D = 50 - 10(2.5) = 50 - 25 = 25$

The equilibrium quantity of plates exchanged in this market is 25.

c. What is the consumer surplus in this market, in units of cups of coffee?

The consumer surplus (CS) is equal to the area of below the demand curve and above the equilibrium price. That is, in this case it is equal to the area of a triangle with height $\frac{5}{2}$ and base 25. The consumer surplus is thus:

$$\frac{1}{2} \cdot \frac{5}{2} \cdot 25 = \frac{125}{4} = 31.25$$

The consumer surplus is equal to 31.25 cups of coffee.

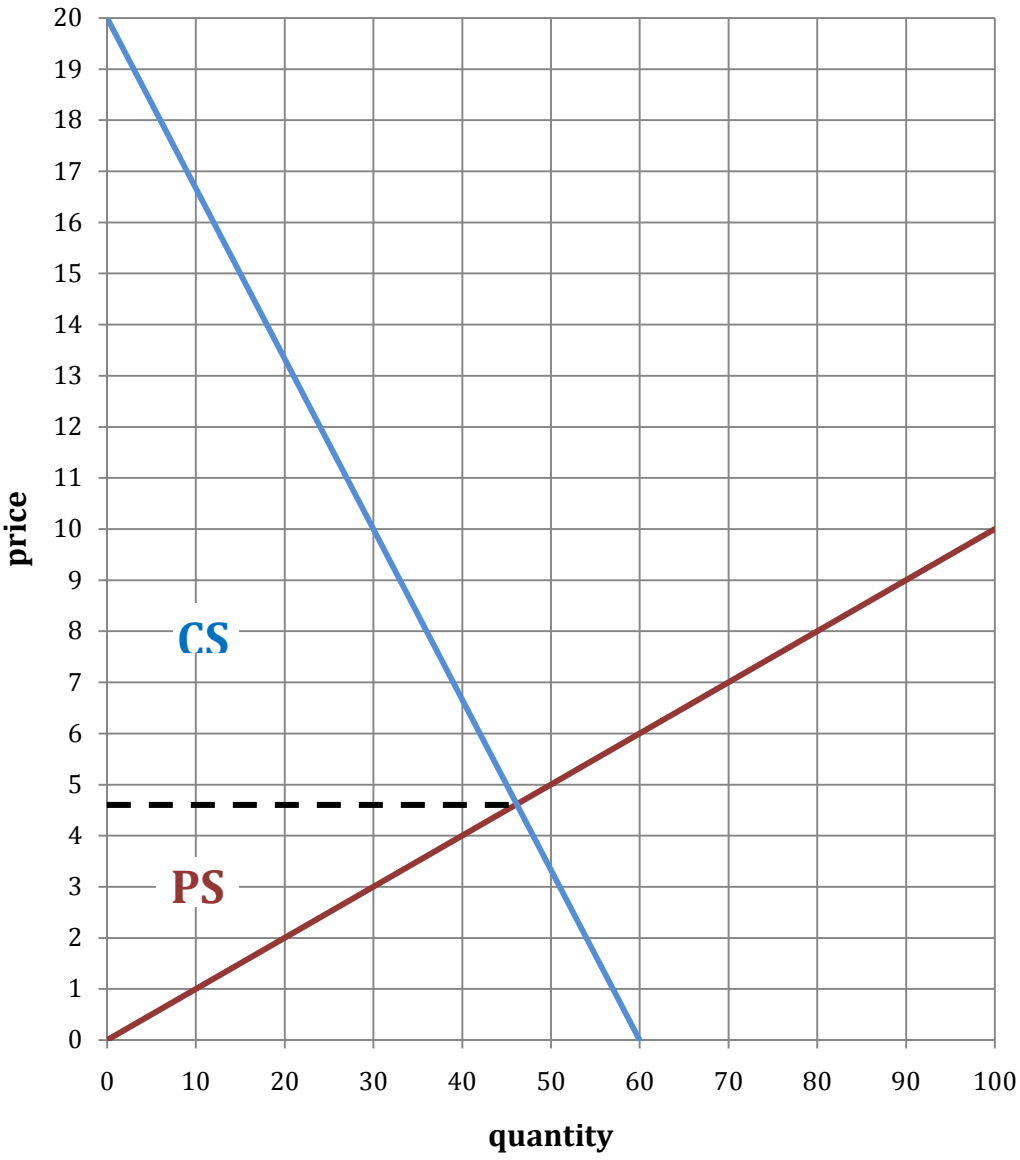
d. What is the producer surplus in this market, in units of cups of coffee?

The producer surplus (PS) is equal to the area of above the supply curve and below the equilibrium price. That is, in this case it is equal to the area of a triangle with height $\frac{5}{2}$ and base 25. The producer surplus is thus:

$$\frac{1}{2} \cdot \frac{5}{2} \cdot 25 = \frac{125}{4} = 31.25$$

The producer surplus is equal to 31.25 cups of coffee.

5. Like (1), but with a different consumer demand: if plates cost 20-cups-of-coffee, nobody wants to buy any. Each 1-cup-of-coffee reduction in the price of plates leads consumers to want to buy an additional three plates.



a. What is the market equilibrium price of plates in this market?

The quantity supplied is given by the equation: $Q_S = 10P$

The quantity demanded is given by the equation: $Q_D = 60 - 3P$

The equilibrium price, P^*, is the price at which the quantity supplied equals the quantity demanded—where $Q_S = Q_D$. Thus:

$$10P^* = 60 - 3P^* \Rightarrow 13P^* = 60 \Rightarrow P^* = \frac{60}{13} = 4\frac{8}{13}$$

The equilibrium price of plates in this market is $4\frac{8}{13}$ cups of coffee.

b. What is the market equilibrium quantity of plates exchanged in this market?

At the equilibrium price, the equilibrium quantity of plates exchanged is given by either the quantity supplied or the quantity demanded: by definition the market equilibrium occurs where these two are the same. Given the equilibrium price found in part a, we have:

$Q_S = 10(\frac{60}{13}) = \frac{600}{13} = 46\frac{2}{13}$

$Q_D = 60 - 3(\frac{60}{13}) = \frac{780}{13} - \frac{180}{13} = \frac{600}{13} = 46\frac{2}{13}$

The equilibrium quantity of plates exchanged in this market is $46\frac{2}{13}$.

c. What is the consumer surplus in this market, in units of cups of coffee?

The consumer surplus (CS) is equal to the area of below the demand curve and above the equilibrium price. That is, in this case it is equal to the area of a triangle with height $20 - \frac{60}{13} = \frac{260}{13} - \frac{60}{13} = \frac{200}{13}$ and base $\frac{600}{13}$. The consumer surplus is thus:

$\frac{1}{2} \cdot \frac{200}{13} \cdot \frac{600}{13} = \frac{60{,}000}{169} = 355\frac{5}{169}$

The consumer surplus is equal to $355\frac{5}{169}$ cups of coffee.

d. What is the producer surplus in this market, in units of cups of coffee?

The producer surplus (PS) is equal to the area of above the supply curve and below the equilibrium price. That is, in this case it is equal to the area of a triangle with height $\frac{60}{13}$ and base $\frac{600}{13}$. The producer surplus is thus:

$\frac{1}{2} \cdot \frac{60}{13} \cdot \frac{600}{13} = \frac{18{,}000}{169} = 106\frac{86}{169}$

The producer surplus is equal to $106\frac{86}{169}$ cups of coffee.

6. Which of the markets in questions 1 through 5 do you think does the most to advance human happiness? Explain your answer.

The market in question 5 does the most to advance human happiness, since it implies the largest total welfare gains. That is, the sum of consumer surplus and producer surplus is highest in the market in question 5. Consider the following table, where all values are expressed in cups of coffee:

market	consumer surplus	producer surplus	total welfare
1	125	125	250
2	0	125	125
3	$111\frac{1}{9}$	$55\frac{5}{9}$	$166\frac{2}{3}$
4	31.25	31.25	62.5
5	$355\frac{5}{169}$	$106\frac{86}{169}$	$461\frac{91}{169}$

Now consider a simpler version of this economy with only three people—Dharma, Egbert, and Greg—who are each able to produce any of the three commodities (coffee, yoga lessons, and ceramic plates). Their abilities to produce the three commodities in one day are given by the following table:

	Coffee	Yoga	Plates
Dharma	2	5	4
Egbert	2	1	3
Greg	9	1	5

That is, in a day Dharma can produce a maximum of 2 cups of coffee **or** 5 yoga lessons **or** 4 plates **or** some linear combination thereof (such as 1 cup of coffee, 0 yoga lessons, and 2 plates).

7. Suppose that we are going to specialize—have each person produce one and only one of the commodities. Decide which of the three commodities should be produced by which of the three people, and explain your reasoning.

   At first glance it seems clear that Dharma should produce yoga lessons and Greg should produce coffee, since each of them is much better at producing those respective goods than their peers.

   However, Egbert is not the best at producing anything. Greg can make more coffee, Dharma can produce more yoga lessons, and either of them can make more ceramic plates than Egbert. Another way to say this is that Egbert doesn't have an *absolute advantage* in producing any of the goods. Nevertheless, look at the opportunity costs for making plates:

   Opportunity costs of making a ceramic plate:

Dharma:	**(0.5 cups of coffee)**	**or**	**1.25 yoga lessons**
Egbert:	$\frac{2}{3}$ **cups of coffee**	**or**	$\frac{1}{3}$ **yoga lessons**
Greg:	**1.8 cups of coffee**	**or**	**(0.2 yoga lessons)**

   Assuming that Dharma will be specializing in yoga lessons and that Greg will be specializing in coffee, Egbert has the *comparative advantage* in making plates: his opportunity costs for doing so are lower than for Dharma or Greg. (Since Dharma will be producing yoga lessons, that's the relevant opportunity cost to compare, likewise for Greg and coffee.) Thus Egbert should specialize in making ceramic plates.

   A separate and very legitimate concern is that the value of these goods depends on their price. If plates are substantially more valuable than coffee Greg should switch to specializing in plates. Or if coffee is much more valuable than plates, Egbert should produce coffee. In other words, there is not enough information given in this question to really determine the optimal production allocation—Dharma, Egbert, and Greg need to know the relative values of these commodities as given by their market prices. This importance of this issue is addressed somewhat in questions 8 and 9 below.

8. Suppose that we take coffee to be our standard of value. Quote the prices of yoga lessons and of ceramic plates in terms of cups of coffee, and suppose that a yoga lesson is worth cu2 (2 cups of coffee) and a ceramic plate is worth cu1 (1 cup of coffee). Does this change your view of the production allocation you had arrived at for question 7? Why or why not?

   Given the prices of yoga lessons and ceramic plates, the *value* of what Dharma, Egbert, and Greg can produce can be expressed in the equivalent number of cups of coffee as follows:

	Coffee	Yoga	Plates
Dharma	**2**	**(10)**	**(4)**
Egbert	**2**	**(2)**	**(3)**
Greg	**9**	**(2)**	**(5)**

   Dharma, Egbert, and Greg collectively produce the most value (in terms of cups of coffee) if Dharma produces yoga lessons, Egbert produces ceramic plates, and Greg produces coffee. That is, the production allocation is the same as that seen in question 7. If any other assignment schedule were used, the total value produced daily would be lower, which would be inefficient.

9. Suppose that we take coffee to be our standard of value. Quote the prices of yoga lessons and of ceramic plates in terms of cups of coffee, and suppose that a yoga lesson is worth cu2 (2 cups of coffee) and a ceramic plate is also worth cu2 (2 cups of coffee). Does this change your view of the best production allocation you had arrived at for questions 7 and 8? Why or why not?

   Given the prices of yoga lessons and ceramic plates, the *value* of what Dharma, Egbert, and Greg can produce can be expressed in the equivalent number of cups of coffee as follows:

	Coffee	Yoga	Plates
Dharma	**2**	**(10)**	**(8)**
Egbert	**2**	**(2)**	**(6)**
Greg	**9**	**(2)**	**(10)**

   Dharma, Egbert, and Greg collectively produce the most value (in terms of cups of coffee) if Dharma produces yoga lessons, Egbert produces ceramic plates, and Greg also produces ceramic plates. While this production allocation produces cu26 worth of goods, any other allocation will produce strictly less value in terms of cups of coffee.

   In contrast to the production allocation found in questions 7 and 8, this new allocation has both Egbert and Greg making ceramic plates. Coffee is so cheap (i.e., it is not valued in the market) that nobody should be making any of it.

   This is seen all the time in the real economies: some potential products are simply left unmade because there is greater value in making other products.