Problem Set #6 Solutions

1. Recall the rule of 72 (see the solutions for Problem Set 1), and recall that sustainable annual consumption spending per capita in the United States today is about $30,000 a year. In those scenarios in which global warming does not fully destabilize the climate or world politics, in which technological change continues at its present pace, in which the world population peaks at 10 billion around 2050, and in which we avoid blowing all of ourselves up with nuclear weapons, we are looking forward to a per capita GDP and consumption growth rate in the rich regions of the world of about 1.8% per year for the foreseeable future. Suppose that these scenarios turn out to be true:

   a. What will be the level of consumption per capita in the United States in 2090?

      $\frac{72}{1.8} = 40$

      Thus consumption spending per capita per year in the United States will be expected to double in 40 years. In 80 years it will double twice, so under these assumptions it will be:

      $2^2 \cdot (\$30,000)$ **= $120,000/year**

   b. What will be the level of consumption per capita in the United States in 2210?

      Since $\frac{200}{40}$ **= 5, over 200 years consumption spending per capita per year in the United States will be expected to double five times, so in 2210 its level will be expected to be:**

      $2^5 \cdot (\$30,000)$ **= $960,000/year**

   c. What do you think life would be like in a society as rich as this scenario suggests that the United States will be by 2210?

      Your guess is probably as good as anyone's. This is indeed the realm of pure speculation. Could anyone from 1810 have begun to envision the world of 2010?

      It might be expected that a great many members of society would be willing to trade income for greater amounts of leisure time, implying that recreation, tourism, and entertainment would be much more important parts of the economy.

      More pessimistically, it may be that a large share of income will need to be devoted to mitigating the effects of climate change.

2. Consider a monopoly seller of tickets to classic Japanese movies. The cost of showing the movie to an extra customer is zero. The quantity demanded for tickets is given by: $Q_D = 1500 - 100P$, where Q_D is the number of tickets that will sell at price P and where P is given in dollars.

Note that $Q_D = 1500 - 100P \Rightarrow P = 15 - 0.01Q_D$.

a. What is the price that the monopolist should charge if it wants to maximize profits?

For any firm profit is maximized where marginal revenue equals marginal cost: $MR = MC$.

Total revenue is price times quantity sold: $TR = P \cdot Q_D = (15 - 0.01Q_D) \cdot Q_D = 15Q_D - 0.01{Q_D}^2$

Marginal revenue is the derivative of total revenue: $MR = \frac{dTR}{dQ_D} = 15 - 0.02Q_D$

Marginal revenue equals marginal cost (zero in this case) where:

$15 - 0.02Q_D = 0 \quad \Rightarrow \quad Q_D = 750$

The profit-maximizing quantity is 750. Given this, the price the monopolist will charge is given by the demand curve: $P = 15 - 0.01(750) = 7.5$

The price that maximizes the monopolist's profit is $7.50.

b. What is the quantity of tickets sold at that price?

750 tickets are sold at the profit-maximizing price (see part a above).

c. What is the consumer surplus?

The consumer surplus (CS) is the area above under the demand curve and above the market price. In this case it is equal to the area of a triangle of height (15 – 7.5) and base 750:

$\frac{7.5 \cdot 750}{2} = 2812.5$

The consumer surplus is $2,812.50.

d. What is the monopolist's profit?

The monopolist's profit is its total revenue minus its total cost, but in this case the monopolist's cost is zero. Its total revenue is: $TR = P \cdot Q_D = 7.5 \cdot 750 = 5625$.

That is, the monopolist's profit is $5,625.

e. Suppose that PDC imposes a price ceiling after deciding that it is unfair to make anybody pay more than $4 a ticket for a movie. What is the consumer surplus? What is the monopolist's profit?

The quantity of tickets sold at the price ceiling of $4 is given by the demand function:

$Q_D = 1500 - 100(4) = 1100$

The consumer surplus (CS) is equal to the area of a triangle of height (15 – 4) and base 1100:

$\frac{11 \cdot 1100}{2} = 6050$

The consumer surplus is $6,050.

Since its total costs are zero, the monopolist's profit is given by its total revenue:

$TR = P \cdot Q_D = 4 \cdot 1100 = 4400$

That is, the monopolist's profit is $4,400.

3. Continue with the situation as in question 2: a monopoly seller of tickets to classic Japanese movies. The cost of showing the movie to an extra customer is zero. The demand curve for tickets is: Q_D = 1500 - 100P, where Q_D is the number of tickets that will sell at price P and where P is given in dollars. But now suppose that there is a fixed cost of $5,000 to obtain the movie and a projector.

Note that $Q_D = 1500 - 100P \Rightarrow P = 15 - 0.01Q_D$.

a. What is the price that the monopolist should charge if it wants to maximize profits?

For any firm profit is maximized where marginal revenue equals marginal cost: $MR = MC$.

Total revenue is price times quantity sold: $TR = P \cdot Q_D = (15 - 0.01Q_D) \cdot Q_D = 15Q_D - 0.01{Q_D}^2$

Marginal revenue is the derivative of total revenue: $MR = \frac{dTR}{dQ_D} = 15 - 0.02Q_D$

Marginal revenue equals marginal cost (zero in this case) where:

$15 - 0.02Q_D = 0 \quad \Rightarrow \quad Q_D = 750$

The profit-maximizing quantity is 750. Given this, the price the monopolist will charge is given by the demand curve: $P = 15 - 0.01(750) = 7.5$

The price that maximizes the monopolist's profit is $7.50.

b. What is the quantity of tickets sold at that price?

750 tickets are sold at the profit-maximizing price (see part a above).

c. What is the consumer surplus?

The consumer surplus (CS) is the area above under the demand curve and above the market price. In this case it is equal to the area of a triangle of height (15 – 7.5) and base 750:

$\frac{7.5 \cdot 750}{2} = 2812.5$

The consumer surplus is $2,812.50.

d. What is the monopolist's profit?

The monopolist's profit is its total revenue minus its total cost:

$\pi = TR - TC = P \cdot Q_D - TC = 7.5 \cdot 750 - 5000 = 625.$

That is, the monopolist's profit is $625.

e. Suppose that PDC imposes a price ceiling after deciding that it is unfair to make anybody pay more than $4 per ticket for a movie. What is the consumer surplus? What is the monopolist's profit?

The quantity of tickets sold at the price ceiling of $4 is given by the demand function:

$Q_D = 1500 - 100(4) = 1100$.

The consumer surplus (CS) is equal to the area of a triangle of height (15 – 4) and base 1100:

$\frac{11 \cdot 1100}{2} = 6050$

The consumer surplus is $6,050.

The monopolist's profit is given by its total revenue minus its total cost:

$\pi = TR - TC = P \cdot Q_D - TC = 4 \cdot 1100 - 5000 = -600.$ **The monopolist's profit is -$600.**

4. Continue with the situation as in question 2: a monopoly seller of tickets to classic Japanese movies. The cost of showing the movie to an extra customer is zero. But now let us consider a variety of demand curves:

a. For $Q_D = 2000 - 100P$: What is the price that the monopolist should charge if it wants to maximize profits? What is the quantity of tickets sold at that price? What is the consumer surplus? What is the monopolist's profit?

Note that $Q_D = 2000 - 100P \Rightarrow P = 20 - 0.01Q_D$.

Total revenue is price times quantity sold: $TR = P \cdot Q_D = (20 - 0.01Q_D) \cdot Q_D = 20Q_D - 0.01{Q_D}^2$

Marginal revenue is the derivative of total revenue: $MR = \frac{dTR}{dQ_D} = 20 - 0.02Q_D$

Marginal revenue equals marginal cost (zero in this case) where:

$20 - 0.02Q_D = 0 \quad \Rightarrow \quad Q_D = 1000.$

The profit-maximizing quantity of tickets sold is 1000. Given this, the price the monopolist will charge is given by the demand curve: $P = 20 - 0.01(1000) = 10$

The price that maximizes the monopolist's profit is $10.

The consumer surplus (CS) is the area above under the demand curve and above the market price. In this case it is equal to the area of a triangle of height (20 – 10) and base 1000:

$\frac{10 \cdot 1000}{2} = 5000$ That is, the consumer surplus is $5,000.

The monopolist's profit is its total revenue minus its total cost:

$\pi = TR - TC = P \cdot Q_D - TC = 10 \cdot 1000 - 0 = 10,000.$ That is, the monopolist's profit is $10,000.

b. For $Q_D = 1000 - 200P$: What is the price that the monopolist should charge if it wants to maximize profits? What is the quantity of tickets sold at that price? What is the consumer surplus? What is the monopolist's profit?

Note that $Q_D = 1000 - 200P \Rightarrow P = 5 - 0.005Q_D$.

Total revenue is price times quantity sold: $TR = P \cdot Q_D = (5 - 0.005Q_D) \cdot Q_D = 5Q_D - 0.005{Q_D}^2$

Marginal revenue is the derivative of total revenue: $MR = \frac{dTR}{dQ_D} = 5 - 0.01Q_D$

Marginal revenue equals marginal cost (zero in this case) where:

$5 - 0.01Q_D = 0 \quad \Rightarrow \quad Q_D = 500.$

The profit-maximizing quantity of tickets sold is 500. Given this, the price the monopolist will charge is given by the demand curve: $P = 5 - 0.005(500) = 2.5$

The price that maximizes the monopolist's profit is $2.50.

The consumer surplus (CS) is the area above under the demand curve and above the market price. In this case it is equal to the area of a triangle of height (5 – 2.5) and base 500:

$\frac{2.5 \cdot 500}{2} = 625$ That is, the consumer surplus is $625.

The monopolist's profit is its total revenue minus its total cost:

$\pi = TR - TC = P \cdot Q_D - TC = 2.5 \cdot 500 - 0 = 1250.$ That is, the monopolist's profit is $1,250.

c. For Q_D = 1500 – 150P: What is the price that the monopolist should charge if it wants to maximize profits? What is the quantity of tickets sold at that price? What is the consumer surplus? What is the monopolist's profit?

Note that $Q_D = 1500 - 150P \Rightarrow P = 10 - \frac{1}{150}Q_D$.

Total revenue is price times quantity sold: $TR = P \cdot Q_D = \left(10 - \frac{1}{150}Q_D\right) \cdot Q_D = 10Q_D - \frac{1}{150}{Q_D}^2$

Marginal revenue is the derivative of total revenue: $MR = \frac{dTR}{dQ_D} = 10 - \frac{1}{75}Q_D$

Marginal revenue equals marginal cost (zero in this case) where:

$10 - \frac{1}{75}Q_D = 0 \quad \Rightarrow \quad Q_D = 750.$

The profit-maximizing quantity of tickets sold is 750. Given this, the price the monopolist will charge is given by the demand curve: $P = 10 - \frac{1}{150}(750) = 5$

The price that maximizes the monopolist's profit is $5.

The consumer surplus (CS) is the area above under the demand curve and above the market price. In this case it is equal to the area of a triangle of height (10 –5) and base 750:

$\frac{5 \cdot 750}{2} = 1875$ **That is, the consumer surplus is $1,825.**

The monopolist's profit is its total revenue minus its total cost:

$\pi = TR - TC = P \cdot Q_D - TC = 5 \cdot 750 - 0 = 3750.$ **That is, the monopolist's profit is $3,750.**

d. For Q_D = 2000 – 500P: What is the price that the monopolist should charge if it wants to maximize profits? What is the quantity of tickets sold at that price? What is the consumer surplus? What is the monopolist's profit?

Note that $Q_D = 2000 - 500P \Rightarrow P = 4 - 0.002Q_D$.

Total revenue is price times quantity sold: $TR = P \cdot Q_D = (4 - 0.002Q_D) \cdot Q_D = 4Q_D - 0.002{Q_D}^2$

Marginal revenue is the derivative of total revenue: $MR = \frac{dTR}{dQ_D} = 4 - 0.004Q_D$

Marginal revenue equals marginal cost (zero in this case) where:

$4 - 0.004Q_D = 0 \quad \Rightarrow \quad Q_D = 1000.$

The profit-maximizing quantity of tickets sold is 1000. Given this, the price the monopolist will charge is given by the demand curve: $P = 4 - 0.002(1000) = 2$

The price that maximizes the monopolist's profit is $2.

The consumer surplus (CS) is the area above under the demand curve and above the market price. In this case it is equal to the area of a triangle of height (4 – 2) and base 1000:

$\frac{2 \cdot 1000}{2} = 1000$ **That is, the consumer surplus is $1,000.**

The monopolist's profit is its total revenue minus its total cost:

$\pi = TR - TC = P \cdot Q_D - TC = 2 \cdot 1000 - 0 = 2000.$ **That is, the monopolist's profit is $2,000.**

5. Suppose that we have a lake that is a little more than a mile around—2100 yards to be precise—and every day there are consumers who might want to rent boats evenly spaced around the lake: one consumer per yard. Periodically around the lake there are boat-rental shacks. People have a disutility of 1 cent per yard that they must walk to rent a boat, and have a reservation price in total cash paid plus disutility of walking of $10. That is, the amount of cash a potential customer is willing to spend on renting a boat is $10 minus $0.01 per yard walked to arrive at the boat-rental shack:

   a. Suppose that you run the only boat-rental shack on the lake and you charge $5. From how far away do people come to rent boats?

      People will come if they have can get a positive (or zero) surplus from walking to your shack and renting a boat. If the charge for renting the boat is $5, the surplus for each person around the lake will be $10 – $5 – $0.01*d* = $5 – $0.01*d*, where *d* is the distance walked to the shack in yards. This expression is nonnegative when:

      $\mathbf{5 - 0.01}\boldsymbol{d} \geq \mathbf{0} \quad \Rightarrow \quad \mathbf{5} \geq \mathbf{0.01}\boldsymbol{d} \quad \Rightarrow \quad \mathbf{500} \geq \boldsymbol{d}$

      That is, people will have positive or zero (but not negative) surplus if they need to walk 500 or fewer yards to the shack, so people will come from 500 yards away to rent boats.

   b. Suppose that you run the only boat-rental shack on the lake and you charge $5. How many boats do you rent?

      As seen above, people will come from 500 yards away to your shack. But since people will come from both directions around the lake, there will be a total of 1000 people coming to the shack.

      Well, 999, 1000, or 1001 depending on what assumptions we make about what the two consumers 500 yards away in each direction will choose to do. They make zero surplus by either renting or not renting. Assume that if potential customers are indifferent between renting and not renting they will choose to buy.

      1000 boats will be rented per day.

   c. Suppose that you run the only boat-rental shack on the lake and you charge $5 and your costs are $2,000/day no matter how many boats you rent. How much money do you make?

      The revenue from renting 1000 boats at $5 each is $5,000. The profit is revenue minus cost: $5,000 – $2,000 = $3,000 per day.

   d. Suppose that you run the only boat-rental shack on the lake and you charge $5. How much consumer surplus do your customers reap?

      The average renter walks 250 yards, so the average consumer surplus is $2.50. With 1000 consumers, the total consumer surplus will be $2,500 per day.

6. Extending question 5, suppose that anybody who wants to can open up a boat-rental shack. The cost to *each* boat-rental shack is $2,000/day no matter how many boats it rents.

For simplicity, assume symmetry. That is, assume that any shacks will always be evenly spaced and that each shack will charge the same price.

a. In equilibrium, how many boat rental shacks do you think there will be?

If the boat-rental shacks are making positive profit, additional entrepreneurs will want to enter the market and set up shacks. On the other hand, if the shacks are making negative profit, one or more will want to exit the market.

In equilibrium, no entrepreneur will want to enter the market, and no existing boat shack will want to exit the market. Additional shacks will enter the market until there are no positive profits to reap.

No matter the number of shacks, a potential customer will walk to the nearest shack and rent a boat if: $10 - P - 0.01d \geq 0 \;\Rightarrow\; d \leq \frac{10}{0.01} - \frac{P}{0.01} \;\Rightarrow\; d \leq 1000 - 100P$

That is, the maximum distance from which a potential customer will walk is 1000 – 10*P*. Since there are customers at every yard and since customers come from both sides of the shack, the demand curve faced by the shack is:

$$Q_D = 2(1000 - 100P) = 2000 - 200P \;\Rightarrow\; P = 10 - 0.005Q_D$$

As seen in question 5, one shack will be able to attract 1000 customers with a price of $5. It turns out that $5 is the profit-maximizing price for one shack to charge.

Total revenue is price times quantity sold: $TR = P \cdot Q_D = (10 - 0.005Q_D) \cdot Q_D = 10Q_D - 0.005{Q_D}^2$

Marginal revenue is the derivative of total revenue: $MR = \frac{dTR}{dQ_D} = 10 - 0.01Q_D$

Marginal revenue equals marginal cost (zero in this case) where:

$$10 - 0.01Q_D = 0 \;\Rightarrow\; Q_D = 1000 \;\Rightarrow\; P = 10 - 0.005(1000) = 5$$

Also seen in question 5, one shack will make positive profit: $3,000 per day.

Two shacks evenly spaced around the lake and charging the same price can attract a maximum of 1050 customers each, but as seen in the case with one shack, each will maximize its profit by charging $5 and attracting 1000 customers. This yields a revenue of $5,000 per day and a profit of $3,000 per day for each shack.

Three shacks evenly spaced around the lake and charging the same price will attract at most 2100/3 = 700 customers each. The profit-maximizing price for each of these shacks to charge is the price that attracts exactly 700 customers (leaving the very farthest customers—350 yards away from a shack in either direction—exactly indifferent between renting or not renting a boat). $P = 10 - 0.005(700) = 6.5$. **This yields a revenue of $6.50 · 700 = $4,550 per shack per day and a profit of $2,550 per shack per day.**

Note that if the three shacks charge a lower price than $6.50, none will attract more than 700 since they are assumed to charge the same price, and this would only lower their revenue. If the three shacks charged more than $6.50 they would make more per customer but they would attract fewer customers. For example, if they charged $6.51 they would each attract 698 customers and make $4,543.98 in revenue, which is less revenue than charging $6.50.

Following this pattern—that for *n* shacks (where *n* is greater than 2) the profit-maximizing price is the price that will attract exactly 2100/*n* customers—we can construct a table of the number of customers, the profit-maximizing price, the revenue to each shack, and the profit to each shack:

Number of shacks	Profit-maximizing quantity	Profit-maximizing price	Revenue of each shack	Profit of each shack
n	$Q_n{}^* = 2100/n$ if $n > 2$; 1000 otherwise	$P_n{}^* = 10 - 0.005Q_n{}^*$	$TR_n = P_n{}^* \cdot Q_n{}^*$	$\pi_n = TR_n - 2000$
1	1000	$ 5.0000	$ 5,000.00	$ 3,000.00
2	1000	5.0000	5,000.00	3,000.00
3	700	6.5000	4,550.00	2,550.00
4	525	7.3750	3,871.88	1,871.88
5	420	7.9000	3,318.00	1,318.00
6	350	8.2500	2,887.50	887.50
7	300	8.5000	2,550.00	550.00
8	262.5	8.6875	2,280.47	280.47
9	233.33	8.8333	2,061.11	61.11
10	210	8.9500	1,879.50	-120.50

From the table it is easy to see that each shack will enjoy positive profit if there are nine shacks or fewer, but each will make negative profit if there are ten or more.

Shacks will enter until there are nine of them, but realizing that a tenth will lead to negative profit for all of them—including the new entrant—no one will want to enter as the tenth shack. Therefore, in equilibrium there will be nine shacks.

b. In equilibrium, how much in profit will the entrepreneurs who establish the boat-rental shacks make?

As seen in the table in part a above, in equilibrium each shack will make $61.11 in profit per day.

c. In equilibrium, how much in consumer surplus will the customers reap?

With nine shacks the shacks are $233\frac{1}{3}$ yards apart, and the average customer has to walk a quarter of that distance: $58\frac{1}{3}$ yards. In dollars, the average consumer surplus is:

$$10 - 8\frac{5}{6} - \frac{1}{100} \cdot 58\frac{1}{3} = \frac{7}{6} - \frac{175}{300} = \frac{7}{6} - \frac{7}{12} = \frac{7}{12}$$

The total consumer surplus is thus: $\frac{7}{12} \cdot 2100 = 1225.$

In equilibrium the customers reap $1225 in consumer surplus per day.

d. If you were running PDC and were in charge of handing out a fixed number of licenses to people to establish boat-rental shacks, how many licenses would you hand out?

Note that the total consumer surplus when there are *n* shacks can also be written as:

$$CS_n = \frac{Q_n^* \cdot (10 - P_n^*)}{2} n$$

Using this, we can construct a new table to assess the welfare effects of the various numbers of shacks:

Number of shacks	Total consumer surplus	Total profit	Total welfare
n	$CS_n = \frac{Q_n^* \cdot (10 - P_n^*)}{2} n$	$\Pi_n = \pi_n \cdot n$	$TW_n = CS_n + \Pi_n$
1	$ 2,500.00	$ 3,000.00	$ 5,500.00
2	5,000.00	6,000.00	11,000.00
3	3,675.00	7,650.00	11,325.00
4	2,756.25	7,487.50	10,243.75
5	2,205.00	6,590.00	8,795.00
6	1,837.50	5,325.00	7,162.50
7	1,575.00	3,850.00	5,425.00
8	1,378.13	2,243.75	3,621.88
9	1,225.00	550.00	1,775.00
10	1,102.50	-1,205.00	-102.50

The optimal number of licenses depends on your goals. If you want to maximize consumer surplus, then you should issue two licenses. However, if you would like to maximize either profit or total welfare, you should issue three licenses.

Note that the optimal number of shacks (either two or three) is much smaller than the equilibrium number of shacks. The moral of the analogy is that there is very often an overprovision of variety in markets with monopolistic competition.